# Performance Analysis of EWMA Controllers Subject to Metrology Delay

Ming-Feng Wu, Chien-Hua Lin, David Shan-Hill Wong, Shi-Shang Jang, and Sheng-Tsaing Tseng

Abstract-Metrology delay is a natural problem in the implementation of advanced process control scheme in semiconductor manufacturing systems. It is very important to understand the effect of metrology delay on performance of advanced process control systems. In this paper, the influences of metrology delay on both the transient and asymptotic properties of the product quality are analyzed for the case when a linear system with an initial bias and a stochastic autoregressive moving average (ARMA) disturbance is under an exponentially weighted moving average (EWMA) run-to-run control. Tuning guidelines are developed based on the study of numerical optimization results of the analytical closed-loop output response. In addition, effective metrology delay of a variable time delay system is analyzed based on the resampling technique implemented to a randomized time delay system. A virtual metrology technique is a possible solution to tackle the problem of metrology delay. The tradeoff between additional error of virtual metrology and reduction in time delay is studied. The results are illustrated using an example of control of the tungsten deposition rate in a tungsten chemical-vapor deposition reactor. The basic conclusion is that metrology delay is only important for processes that experience nonstationary stochastic disturbance. In such a case, use of virtual metrology is justified if the error of the virtual metrology method is less than the error caused by stochastic process noise. The accuracy of the virtual metrology noise with respect to the traditional metrology is not critical, provided that the error due to metrology is much less than that due to process disturbances.

*Index Terms*—Asymptotic mean square error (AMSE), exponentially weighted moving average (EWMA) controller, resampling, run-to-run (RtR) control, transient.

## I. INTRODUCTION

**S** EMICONDUCTOR manufacturing has always been an industry of high capital investment. It is fast transforming into an industry of marginal profits. Advanced process control (APC) has become one of the key technologies for companies to produce products with stringent quality assurance and remain competitive. One of the important elements of APC is run-to-run (RtR) control, which can be used to eliminate initial recipe bias, process shifts, and patterned disturbance [1], [2]. Various RtR control algorithms have been proposed and many aspects of their performance and applications have been studied

M.-F. Wu, D. S.-H. Wong, and S.-S. Jang are with the Department of Chemical Engineering, National Tsing-Hua University, Hsin Chu, Taiwan, R.O.C. (e-mail: ssjang@mx.nthu.edu.tw).

C.-H. Lin and S.-T. Tseng are with the Department of Statistics, National Tsing-Hua University, Hsin Chu, Taiwan, R.O.C.

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[3]–[5]. One of the issues in the application of RtR control is how metrology delay would affect the performance of the controller.

The effect of metrology delay was first investigated by Box and co-workers [6]. On the other hand, time delay systems have also been substantially studied in traditional process control systems. Stability analyses of linear systems can be determined by allocating the positions of the poles [7]. A RtR control system can be viewed as the discrete extension of continuous time-delay control systems and hence model-based analysis can be implemented [8], [9]. Qin and co-workers [10], [11] investigated the stability analysis of RtR time-delay systems.

In actual plant operations, decision makers at various levels often demand estimates of performances of the controller at different delays to answer the following questions.

- Is the investment in advanced metrology justified? Metrology equipments are very expensive. The cost of reducing metrology delay by adding more metrology tools or the use of advanced tool with on-spot metrology must be justified by prior estimates of how much improvement in process capability can be realized.
- 2) How do we retune the controller parameters if the metrology delay is changed? The actual metrology delay may change due to maintenance of metrology tools or addition of new metrology tools. The guidelines for retuning the RtR controllers to achieve optimal performance would be helpful.
- 3) Can virtual metrology be used? Virtual metrology methods that predict quality characteristics using sensors data of the tool have been developed for process monitoring purposes. However, there are often debates on whether such methods should be used in RtR control. The gain in process capabilities by reducing delays must be weighed against the loss in accuracy of quality measurements by the virtual metrology.
- 4) Moreover, in real applications, the metrology delay is not a constant. The average delay is usually used as the basis of controller development [12]. Do the above guidelines apply in case of variable delays? If yes, what is the effective metrology delay for these systems?

In this paper, the effect of metrology delay on transient and asymptotic performances of an exponentially weighted moving average (EWMA) RtR control on a linear system are derived to answer the above questions. The effect of delay on the ability of the EWMA controller to correct initial bias is evaluated using sum of square error during transient to steady state. In case of

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Fig. 1. Block diagram of an EWMA run-to-run controller.

stochastic disturbances, the effect of time delay on closed-loop plant performance is evaluated based on the asymptotic mean square error (AMSE). The technique of resampling is applied to derive the effective metrology delay.

The rest of this paper is organized as follows. The problem formulation is presented in the next section. Performance analysis of metrology delay for RtR control system and tuning guidelines are provided in Section II. Implementation of virtual metrology will be investigated in Section III. The effect of variable delay is discussed in Section IV. An illustrative example including control of the tungsten deposition rate in a tungsten chemical–vapor deposition (CVD) reactor are given in Section V. Conclusions are presented in Section VI.

#### **II. SYSTEM EQUATION AND CLOSED LOOP RESPONSE**

Let us consider a process with linear input and output relation

$$Y_t = \alpha + \beta X_t + \eta_t \tag{1}$$

with  $Y_t$  being the plant output, and  $X_t$  the control action taken for run t, and  $\alpha$  is the initial bias of process.  $\beta$  is the process gain.  $\eta_t$  is the disturbance input.

Given a process predicted model

$$\tilde{Y}_t = a + bX_t \tag{2}$$

where a and b are model offset and gain parameters estimated for the system, respectively. Suppose that there is a metrology delay d, the measurement available before the tth run is actually the result of run (t - d - 1). Using an EWMA filter, the disturbance is estimated to be

$$\tilde{\eta}_t = \lambda (Y_{t-d-1} - a - bX_{t-d-1}) + (1 - \lambda)\tilde{\eta}_{t-1}.$$
 (3)

Control action is

$$X_t = \frac{\tau - a - \tilde{\eta}_t}{b} \tag{4}$$

where  $\tau$  represents the target. Fig. 1 shows the block diagram of the above algorithm.

Without loss any generality, let  $\tau = 0$ , then the close loop response is found to be

$$Y_t = \left[\frac{1 - (1 - \lambda)z^{-1} - \lambda z^{-(d+1)}}{1 - (1 - \lambda)z^{-1} - (\lambda - \xi\lambda)z^{-(d+1)}}\right] (\alpha - \xi a + \eta_t) = \Gamma_t + W_t$$
(5)

with  $z^{-1}$  being the backshift operator and  $\xi = \beta/b$  represents an uncertainty index for model gain. Using long division, we have

$$\Gamma_t = \left(\sum_{i=1}^t p_i z^{-i}\right) (\alpha - \xi a) \tag{6}$$

$$W_t = \left(\sum_{i=1}^t p_i z^{-i}\right) \eta_t \tag{7}$$

with

$$p_{i} = \begin{cases} 1 & i = 1\\ 0 & i \in [2, d+1]\\ (r+s-1)r^{i-d-2} & i \in [d+2, 2d+2]\\ p_{i-1}r + p_{i-d-1}s & i \in [2d+3, \infty) \end{cases}$$
(8)

and  $r = 1 - \lambda$ ,  $s = \lambda(1 - \xi)$ .

In this paper, the EWMA controller performance is discussed based on the abilities of bias correction (i.e.,  $Y_t = \Gamma_t$ ) and stochastic disturbance rejection ( $Y_t = W_t$ ).

## III. CONTROLLER PERFORMANCE AND TUNING

## A. Bias Correction

First, the problem of bias correction is discussed. If we neglect the stochastic disturbance term  $\eta_t$ , (5) is simplified into

$$Y_t = \left[\frac{1 - (1 - \lambda)z^{-1} - \lambda z^{-(d+1)}}{1 - (1 - \lambda)z^{-1} - (\lambda - \xi\lambda)z^{-(d+1)}}\right] (\alpha - \xi a) = \Gamma_t.$$
(9)

Given the model gain and offset (a and b) and the actual process gain and offset ( $\alpha$  and  $\beta$ ) the model error term ( $\alpha - \xi a$ ) will be a constant. Hence, the closed loop response at sampling instant t is given by

$$\Gamma_t = B_t(\alpha - \xi a) \tag{10}$$

$$B_t = p_0 + p_1 + \cdots p_t \tag{11}$$

with

$$B_t = \begin{cases} 1 & t \in [1,d] \\ 1 + \frac{(r+s-1)(1-r^{t-d-1})}{1-r} & t \in [d+1,2d+1] \\ rB_{t-1} + sB_{t-d-1} & t \in [2d+2,\infty) \end{cases}$$
(12)

Property 1: For an overestimated process gain, i.e.,  $\xi \leq 1$ ,  $B_t$ , given in (12) is a monotonic decreasing sequence. For an underestimated process gain, i.e.,  $\xi > 1$ , there exist some values of  $\lambda$  larger than which the system (12) is oscillatory.

Proof: See the Appendix.



Fig. 2. The effect of run number (a) N = 20 and (b) N = 200 on initial bias correction ability as a function of  $\lambda$  with  $\xi \ge 1$ , d = 4.

The previous properties show, in case the process gain is overestimated  $\xi < 1$ , system (12) is always stable but overdamped. In case of underestimation of process gain ( $\xi > 1$ ), system (12) is very possibly underdamped (oscillatory). However, the performance of underestimated model-based control may have better performance with the same absolute value of modeling error. As  $\xi < 1$ , the response of the closed-loop system behaves as monotonic decreasing. On the other hand, as  $\xi > 1$ , the system behaves oscillatorily.

The ability of the controller to correct an initial bias  $(\alpha - \xi a)$  can be rated by the sum of square error (SSE) due to the initial bias. The following can be easily derived from (5):

(0,1). The effects of  $\lambda$  on SSE with N = 20, and 200 for overestimated gains  $\xi \ge 1$  are shown for d = 4 in Fig. 2.

We found that the optimal value of  $\lambda$  does not change much with different run numbers. Therefore, we will just discuss the effect of delay on optimal values of  $\lambda$  and SSE for the long run case (N = 200). Fig. 3(a) shows that the optimal value of  $\lambda$ decreases as the delay increases. Fig. 3(b) shows the optimal SSEs are near linear function of time delay.

In addition, we analyze the performance of bias correction in the presence of metrology noise, i.e.,  $\eta_t = \nu_t \in N(0, \sigma_{\nu}^2)$  being a Gaussian distributed metrology noise. The expected value of the SSE is given by

$$SSE(\xi, \lambda, d) = \left(\sum_{t=1}^{N} B_t^2\right) (\alpha - \xi a)^2$$
(13)

where N is the total run number of each tool between runs.

Consider a closed-loop system (5) with an overestimated process gain, i.e.,  $\xi < 1$ , and a given time delay d, the following is true.

Property 2: In case of  $\xi \leq 1$ , if  $\lambda_1 > \lambda_2(0 \leq \lambda \leq 1)$ for all t > d, then  $B_t(\xi, \lambda_1, d) \leq B_t(\xi, \lambda_2, d)$ . Therefore,  $SSE(\xi, \lambda_1, d) \leq SSE(\xi, \lambda_2, d)$ .

*Proof:* See the Appendix.

Corollary 1: In case of  $\xi \leq 1$  and  $\lambda \in [0, 1]$ , the optimal value of  $\lambda$  is equal to one  $\min \text{SSE}(\xi, \lambda_1, d) = \text{SSE}(\xi, 1, d)$ .

*Proof:* This corollary is a natural extension of Property 2, since  $0 \leq \lambda \leq 1$ .

In the case of  $\xi \ge 1$ , since the response will be underdamped, a controller that is too active will caused oscillator behavior and inferior SSE. There will be an optimal value of  $\lambda$  in the range

$$E\left[\text{SSE}(\xi,\lambda,d)\right] = \left(\sum_{t=1}^{N} B_t^2\right) (\alpha - \xi a)^2 + \sigma_v^2 \left(\sum_{t=1}^{N} p_t^2\right).$$
(14)

A ratio of metrology noise to the magnitude of error initial bias estimate is defined as  $\Omega = \sigma_{\nu}^2/(\alpha - \xi a)^2$ . Given  $\Omega = 0.2$ , 0.1, optimal values of  $\lambda$  at different values of d and  $\xi$  can be found numerically using (11), (12), and (14) (Fig. 4). Due to metrology noise, the controller should not be fully opened even if the model gain is overestimated. There is little need to change the controller setting as metrology delay increases. As we have seen in the noise-free case, it is advisable that the controller be detuned with increased delay if we assume a model gain that is smaller than the actual process gain  $\xi > 1$ . However, the amount of detuning required when delay increases is less as noise level increases.

Fig. 5 demonstrates the optimal values of E[SSE] at different delays and model uncertainty  $\xi$ . Although the E[SSE] are also near linear functions of time delay as in the noise free case in



Fig. 3. The effects of time delay d on (a) the optimal  $\lambda$  and (b) optimal SSE with  $\xi \ge 1$ , and N = 200.



Fig. 4. The effects of time delay d on the optimal  $\lambda$  (N = 200) for bias correction at different noise to initial bias ratios  $\Omega$ . (a)  $\Omega = 0.1$ . (b)  $\Omega = 0.2$ .

Fig. 3(b), the slope become much less significant as the noise level increases.

## B. Time-Correlated Noise Reduction

Another function of an EWMA RtR controller is to correct for time-correlated disturbance. For simplicity, let us assume that no initial bias exists and that the disturbance consists of an ARMA(1,1) noise, which is commonly used in literature [2], [4], [13] to describe stochastic process disturbances, plus a Gaussian distributed white noise due to metrology

$$\eta_t = \frac{1 - \theta z^{-1}}{1 - \phi z^{-1}} \varepsilon_t + \nu_t. \tag{15}$$

Note that when  $\phi = 1$  the process disturbance becomes a nonstationary process disturbance IMA(1,1); when  $\phi < 1$  the process disturbance is stationary RtR correlated process disturbance. The closed-loop response is given by

$$Y_{t}(\xi, \lambda, d, \theta) = W_{t}$$

$$= \left[ \frac{1 - (1 - \lambda)z^{-1} - \lambda z^{-(d+1)}}{1 - (1 - \lambda)z^{-1} - (\lambda - \xi\lambda)z^{-(d+1)}} \right] \times \left( \frac{1 - \theta z^{-1}}{1 - \phi z^{-1}} \varepsilon_{t} + v_{t} \right)$$
(16)

with  $\varepsilon_t \in N(0, \sigma_{\varepsilon}^2)$   $v_t \in N(0, \sigma_{\nu}^2)$  being zero mean Gaussian distributed random numbers. Using long division,  $W_t$  can be divided into a part that is related to the length of the delay and



Fig. 5. Effects of delay d on optimal E[SSE] (N = 200) for bias correction at different noise to initial bias ratios  $\Omega$ . (a)  $\Omega = 0.1$ . (b)  $\Omega = 0.2$ .

the structure of the RtR correlated disturbance, but independent of the controller; another part is dependent on the tuning of the controller

$$W_t = \left[n_0 + n_1 z^{-1} + \dots + n_d z^{-d} + Q(\lambda, \xi, \phi, \theta, d, z^{-1})\right] \varepsilon_t + \left(\sum_{i=0}^t p_i z^{-i}\right) v_t \quad (17)$$

with

$$n_{i} = \begin{cases} 1 & i = 0\\ (\phi - \theta)\phi^{i-1} & i = 1, d \end{cases}$$
(18)

$$Q(z^{-1}, r, s, \varphi, \theta, d) = \left[r + s - 1 + (\varphi - \theta)\varphi^d\right] z^{-(d+1)} + \frac{A(z^{-1}, r, s, \varphi, \theta, d)}{K(z^{-1}, r, s, \varphi, \theta, d)}$$
$$\equiv z^{-d} \sum_{k=1}^{\infty} q_k z^{-1}$$
(19)

with

$$\begin{aligned} A(z^{-1}, r, s, \varphi, \theta, d) \\ &= \left[ -\theta(r+s-1) + r\varphi^d(\varphi - \theta) \\ &+ (r+s-1+\varphi^{d+1} - \theta\varphi^d)(r+\varphi) \right] z^{-(d+2)} \\ &- \left[ r+s-1 + (\varphi - \theta)\varphi^d \right] r\varphi z^{-(d+3)} \\ &+ s(r+s-1)^{-(2d+2)} \\ &- \varphi s \left[ r+s-1 + (\varphi - \theta)\varphi^d \right] z^{-(2d+3)} \end{aligned} \tag{20}$$

$$K(z^{-1}, r, s, \varphi, \theta, d) \\ &= 1 - [r+\varphi] z^{-1} + \varphi r z^{-2} - s z^{-(d+1)} \\ &+ \varphi s z^{-(d+2)}. \end{aligned}$$

The previous analysis can be inferred directly from the work on controller performance analysis by Harris 1989 [14].

The asymptotic mean square error(AMSE) is given by

$$AMSE(\varphi, \theta, d, \lambda, \xi, \sigma_{\varepsilon}, \sigma_{v}) = \lim_{t \to \infty} \left( \sum_{k=0}^{d} n_{k}^{2}(\varphi, \theta, d) + \sum_{k=1}^{t} q_{k}^{2}(\varphi, \theta, d, \lambda, \xi) \right) \sigma_{\varepsilon}^{2} + \lim_{t \to \infty} \left( 1 + \sum_{k=1}^{t} p_{k}^{2}(\varphi, \theta, d, \lambda, \xi) \right) \sigma_{v}^{2}.$$
(22)

Fig. 6 shows optimal values of  $\lambda$  and AMSE found by solving the equation when  $\varphi = 1$ ,  $\sigma_{\varepsilon}^2 = 1$ , and  $\sigma_{\nu}^2 = 0$ . This corresponds to the case of a nonstationary RtR noise. Also, it is assumed that the process disturbance is much larger than metrology noise. It is found that the AMSE increases linearly with delay but the slope decreases with  $\varphi - \theta$ . Similarly, the optimal 1 needs to be detuned but the amount of detuning required decreases with  $\varphi - \theta$ . Note that when  $\varphi = \theta = 1$ , the disturbance is a white noise and no control is required.

Fig. 7 shows optimal AMSE found by solving the previous equation when  $\varphi = 0.8$ ,  $\sigma_{\varepsilon}^2 = 1$ , and  $\sigma_{\nu}^2 = 0$ . This corresponds to the case of a strongly RtR correlated but stationary noise. It is found that the AMSE levels out and only a sluggish controller is required after a delay of 4.

The numerical analysis leads to an important conclusion: the effect of long metrology delay on controller performance is significant only for a process with nonstationary noise. This conclusion can be supported using the following argument. Since the controller independent part cannot be eliminated, the optimal AMSE is always greater than

$$\begin{aligned} \text{AMSE}_{\text{opt}} \geq \sigma_{\varepsilon}^{2} \left[ \sum_{k=0}^{d} n_{k}^{2}(\varphi, \theta, d) \right] \\ &= \left[ 1 + \frac{(\varphi - \theta)(1 - \varphi^{2d})}{(1 - \varphi^{2})} \right] \sigma_{\varepsilon}^{2} \\ &\to \left[ 1 + d(1 - \theta) \right] \sigma_{\varepsilon}^{2} \quad \varphi \to 1 \\ &\to \left[ 1 + \frac{(\varphi - \theta)}{(1 - \varphi^{2})} \right] \sigma_{\varepsilon}^{2} \quad \varphi < 1, d \gg 1. \end{aligned}$$
(23)



Fig. 6. Effects of delay d on optimal AMSE and optimal  $\lambda$  at different model gain errors with a non-stationary IMA(1,1) noise.

The above lower bound of AMSE is linear with respect to delay only when  $\varphi = 1$ . With  $\varphi < 1$ , the effect of increasing delay become very small at large delays.

case of stationary noise  $\varphi < 1$ , the controller should be turned on only if there is a strong RtR correlation and the delay is no more than two to three runs.

In the case of controller tuning, we found that the EWMA filter must be detuned with increased delay if  $\xi > 1$  in the case of nonstationary noise, i.e.,  $\varphi = 1$ . If  $\xi < 1$  and  $\varphi = 1$ , optimal values of  $\lambda$  are relatively unaffected since we must keep an active controller to result in sufficient manipulative action. In the

# C. Implementation of Virtual Metrology

Use of virtual metrology (VM) techniques have been a very active research topic in the AEC/APC community in semiconductor



Fig. 7. Effects of delay d on optimal AMSE and optimal  $\lambda$  at different model gain errors with a strongly autoregressive, but stationary ARMA(1,1) noise.

manufacturing [15], [16]. A VM tries to predict the product quality  $Y_k$ , using the equipment data of the current run. Hence,

the delay is reduced to zero. However, VM technology is usually subjected to increased inaccuracy compared to actual metrology.



Fig. 8. The benefits of VM as function of the accuracy measure  $\Psi_{\cdot}(a) \sigma_{\varepsilon} = 1, \sigma_{v} = 1.$  (b)  $\sigma_{\varepsilon} = 1, \sigma_{v} = 0.1$ .

Again, let us assume that the plant is subjected to stochastic noise of ARMA(1,1). In the absence of virtual metrology the closed-loop response is given by (16) and the asymptotic mean square error is given by (22). If a VM is implemented, the closed-loop performance is given by setting d = 0 and replacing  $v_t \in N(0, \sigma_{\nu}^2)$  with the VM  $v_{VM,t} \in N(0, \sigma_{VM}^2)$ , which is assumed to be also unbiased but larger  $\sigma_{VM}^2 \ge \sigma_{\nu}^2$ 

$$Y_{\rm VM}(\xi,\lambda,d,\theta) = \left[\frac{1-(1-\lambda)z^{-1}-\lambda z^{-2}}{1-(1-\lambda)z^{-1}-(\lambda-\xi\lambda)z^{-2}}\right] \times \left(\frac{1-\theta z^{-1}}{1-\phi z^{-1}}\varepsilon_t + v_{VM,t}\right). \quad (24)$$

Similarly, an expression for AMSE is obtained as

$$AMSE(\varphi, \theta, 0, \lambda, \sigma_{\varepsilon}, \sigma_{VM}) = \lim_{t \to \infty} \left( \sum_{k=1}^{\infty} q_k^2(\varphi, \theta, 0, r, s) \right) \sigma_{\varepsilon}^2 + \lim_{t \to \infty} \left( 1 + \sum_{k=1}^{\infty} p_k^2(\varphi, \theta, 0, r, s) \right) \sigma_{VM}^2.$$
(25)

Fig. 8(a) demonstrates the ratio of optimal AMSE at different delays d and  $\theta$ , at  $\varphi = 1$ , and  $\xi = 1$ , as a function of  $\Psi = \sigma_{\rm VM}/(\sigma_{\varepsilon} + \sigma_{v})$  with  $\sigma_{\varepsilon} = \sigma_{v} = 1$ . The parameter  $\Psi$ can be regarded as an accuracy measure of the VM. It is clear that VM is very helpful to a nonstationary process even if error of the virtual metrology is significant compared to the process error and metrology error. However, as the autocorrelation characteristics  $\varphi - \theta$  decrease, the benefits become less and less significant. In the extreme case when the disturbance is a white noise, use of VM will be detrimental. In Fig. 8(a), the metrology noise is fairly large compared to random process disturbances  $(\sigma_{\varepsilon} = \sigma_{v} = 1)$ . The accuracy of VM cannot be better than actual metrology, the minimum achievable  $\Psi$  can only starts at 0.5. In Fig. 8(b),  $\sigma_{\varepsilon} = 1 = 10\sigma_v$ , the metrology noise is small compared to process noise. It is possible to develop a VM with noise smaller than the random process disturbances, i.e.,  $\Psi$  can start at 0.1/1.1. The benefits of VM on reducing delays for nonstationary processes will be more significant although they diminish as the RtR autocorrelation of the process noise diminishes. It should, however, be pointed out that in such a case, there will still be significant benefits even if the accuracy of VM is relatively poor compared to actual metrology.

# IV. VARIABLE DELAY

In an actual manufacturing plant, measurement delay is a stochastic variable instead of being fixed. Usually, an effective metrology delay (EMD) is taken as the average of the observed delays. It will be interesting to investigate whether the controller performance calculated using the EMD is indeed equal to the actual plant performance. Let  $m_k$  be a random number generated



Fig. 9. Comparison of resampled and original Poisson distributions at different values of  $\mu$ . (a) Poisson distribution  $\mu = 1$ ; (b) resampled Poisson distribution  $\mu = 1$ ; (c) Poisson distribution  $\mu = 2$ ; (d) resampled Poisson distribution  $\mu = 5$ ; and (f) resampled Poisson distribution  $\mu = 5$ .

at the kth run by a Poisson distribution, and let  $d_k$  are variable metrology delay sequence. However, if the metrology delay of the (k+1)th run is longer than that of the kth run, then the measured data will become out of sequence and cannot be used in feedback. Hence, the actual resampled distribution is given by

$$d_k = \begin{cases} m_{k-1}, & \text{if } m_k \le m_{k-1} + 1\\ m_{k-1} + 1, & \text{otherwise} \end{cases}$$
(26)

In Fig. 9, the resampled distributions are compared with the original Poisson distributions at different values of  $\mu$ , where  $\mu$  is the mean of a Poisson distributed random number. It was found that the tails of the original Possion distribution are truncated and the mode of the distribution will become more populated. Fig. 10 plots the sampled mean and variance of the resampled distribution. Each sampling is made up of 1000 runs and ten



Fig. 10. Mean and variance of the resampled Poission distribution versus  $\mu$  (lines—original mean and variance of Poisson distribution, points resampled mean and variance). (a) Resampled mean. (b) Resampled variance.

different samples were obtained. For the modified Poisson distribution with a parameter  $\mu$ , both the observed mean and variance will be smaller than  $\mu$  and the corresponding variance of the Poisson distribution. Moreover, the distribution of the sampled mean and variance will broaden as  $\mu$  increases.

To study the effect of variable time delay, 100 sequences of 10 000 random numbers are generated using the modified Poisson distribution with a given  $\mu$ . Simulations were carried out using these sequences of random numbers as time delays and an EWMA controller with  $\lambda = 0.3$ . The model gain error is  $\xi = 0.85$ . The results of the last 1000 runs were used to estimate the AMSE and the observed average delay. The procedure was repeated with different values of  $\mu$ . Fig. 11 plotted simulated values of AMSE<sub>N</sub> against observed average delay. It can be found that AMSE calculated by (22) using a fixed effective mean delay provides a good estimate of the average AMSE<sub>N</sub>. However, the actual AMSE obtained have a wide distribution even though the observed effective mean delay showed only small variations.

# V. ILLUSTRATED EXAMPLE: TUNGSTEN CVD PROCESS

Consider a tungsten CVD process as described in [8], [17], where tungsten is deposited onto the wafer by  $H_2$  reduction of  $WF_6$ . Assume it is desired to control the tungsten rate in the reactor with the following model for the deposition rate:

$$\ln(R_w) = c_0 + c_1 \frac{1}{T} + c_2 \ln[H_2]$$
(27)

where  $R_w$  = deposition rate (in Angstrons per minute), T = Temperature (K), and  $[H_2]$  = Partial pressure of hydrogen torr in the reactor.

The objective of this problem is to control the reaction rate by manipulating the reactor temperature and partial pressure of hydrogen subjected to a metrology delay (d = 3). In the following study, we assume  $c_0 = \ln(2 \times 10^8)$ ,  $c_1 = -8800$ ,  $c_2 = 0.5$ . By setting  $y = \ln(R_w)$ , and  $\mathbf{x} = (1/T, \ln[H_2])'$  [17], then y can be



Fig. 11. Simulated AMSE versus observed averaged delay using the modified Poisson distributions.

controlled based on the EWMA algorithm as shown in Fig. 1. Let

$$y_t = \mathbf{x}'\boldsymbol{\beta} + \eta_t \tag{28}$$

where  $\beta = (-8800 \ 0.5)'$ , and the model to characterize the process behavior by

$$\hat{y}_t = \mathbf{x}'\hat{\beta} \tag{29}$$

where  $\hat{\beta} = (-7333 \ 0.7)'$ , and hence the model error  $\xi = (\hat{\beta}'\beta/\hat{\beta}'\hat{\beta}) = 1.2$  [8]. Let the target of deposition rate be 4200 A/min, and  $\tau = \ln(4200)$ . By minimizing the change of the control action **x** [8], the MISO problem can be solved by

$$\mathbf{x}_{n} = \frac{(\tau - \hat{\eta}_{n-1})}{\hat{\beta}'\hat{\beta}}\hat{\beta} + \left(1 - \frac{\hat{\beta}\hat{\beta}'}{\hat{\beta}'\hat{\beta}}\right)\mathbf{x}_{n-1}.$$
 (30)



Fig. 12. Process dynamics with RtR controller for tungsten CVD process. (a) Process response, (b) IMA(1,1) disturbance input, (c) hydrogen pressure, and (d) reactor temperature.



Fig. 13. Optimal AMSE of the tungsten CVD process at difference metrology delay (line is AMSE of equation, symbol is the simulation result).

Let us implement the following IMA(1,1) noise:

$$\eta_t = \frac{1 - 0.5z^{-1}}{1 - z^{-1}} \varepsilon_t + v_t \tag{31}$$



Fig. 14. The benefits of VM as a function of its accuracy  $\Psi$ .

 $\varepsilon_{t,}, v_t \in N(0, 0.1)$ . Fig. 12(a)–(d) shows the deposition rate, disturbance, temperature, and partial pressure of hydrogen, respectively. From Fig. 12(c), we find [H<sub>2</sub>] pressure is unchanged.

This is due to the fact that  $c_2$  is so much smaller than  $c_1$ . So, this case is actually a single in/single out (SISO) problem. The asymptotic mean square error for  $\log(R_w)$  is 0.0292 compared with the forecast value of 0.0289 by (22). Fig. 13 shows the effect of time delay on controller performance through simulation (symbols) and (22). Note that in the original papers [8], [17], some other control schemes are proposed. However, in order to compare with the analytical results, only (27) is implemented.

Fig. 14 shows that the performance of VM in a different noise ratio. We can find that the result is similar to Fig. 8. If the noise ratio  $\Psi$  is more than 0.67, the use of VM is futile. Since in this case  $\sigma_{\varepsilon} = \sigma_v$ , a fairly accurate VM method, i.e.,  $\sigma_{\rm VM} < 1.17\sigma_v$ , is required.

# VI. CONCLUSION

In this paper, the effects of delay on the ability of an EWMA controller to correct initial bias and eliminate RtR correlated errors are studied. Given these analyses, the following answers to the decision-making problems listed in Section I can be provided.

- Is the investment in advanced metrology justified? Spending on additional metrology tool advanced metrology to reduce metrology should be justified only if careful analysis of the disturbance pattern has been carried out. Improvement in process capabilities by reducing metrology delay can be realized only on those processes that demonstrated nonstationary or highly autoregressive disturbances.
- 2) How do we retune the controller parameters if the metrology delay is changed? If we design a conservative controller to begin with, i.e., assuming a large process model gain,  $\xi \leq 1$ , there is no need to detune with increased delay, when the noise is nonstationary (Fig. 6). If we started with an active controller using a small model process gain, the controller needs to be detuned when metrology delay is increased when we have a nonstationary process noise (Fig. 6). When we have a strongly correlated but stationary noise, then the EWMA parameter must be detuned as delay increases regardless of  $\xi$ . However, there is no need to further detune once delay passes 4 to 5 (Fig. 7).
- 3) Can virtual metrology be used? The benefits of VM depend on its accuracy of the VM method and the disturbance structure. VM should only be used on those processes that demonstrated nonstationary or highly autoregressive disturbance. When VM is deployed, it is the accuracy with respect to the magnitude of random process noise that is critical. The accuracy of VM with respect to actual metrology is not important if actual metrology noise is small compared to process disturbances.
- 4) Do the above guidelines apply in case of variable delays? The above guidelines apply in general even if the delays are variable. The mean of the resampling random variable can be viewed as the effective metrology delay. However,

the variations observed in a finite sampling period will be larger.

## APPENDIX

Proof of Property 1: Given that  $0 < \lambda < 1, \xi \leq 1$ , and  $r + s - 1 = -\lambda \xi \leq 0$ , it is obvious from (12) that  $B_t > 0$  and  $B_t \leq B_{t-1}$  for all  $t \leq 2d + 1$ . From (12), when  $t \geq 2d + 2$ 

$$B_{t} - B_{t-1} = p_{t} = rp_{t-1} + sp_{t-d-1}$$
  
=  $r(B_{t-1} - B_{t-2}) + s(B_{t-d-1} - B_{t-d-2})$   
 $\leq 0.$  (A.1)

Hence,  $B_t$  is a monotonic decreasing but positive definite series for all t.

For any  $\xi > 1$ , we can find some value of  $0 < \lambda < 1$ , so that  $B_t < 0$  at t = 2d + 2

$$B_{t} = 1 + \frac{(r+s-1)(1-r^{2d+2-d-1})}{1-r}$$
  
= 1 - \xi \left(1 - (1-\lambda)^{t-d-1}\right) < 0  
\Rightarrow \lambda > 1 - \left(\frac{\xi - 1}{\xi}\right)^{\frac{1}{d+1}}. \quad \text{(A.2)}

Moreover, since the system is stable and  $B_{t\to\infty} = 0$ , we can show that  $B_t$  will not be monotonic decreasing, i.e., the system will exhibit oscillatory behavior.

Proof of Property 2: We have

$$B_{t}(\xi,\lambda_{1},d) - B_{t}(\xi,\lambda_{2},d)$$

$$= 1 + \frac{(r_{1} + s_{1} - 1)(1 - r_{1}^{t-d-1})}{1 - r_{1}} - 1$$

$$- \frac{(r_{2} + s_{2} - 1)(1 - r_{2}^{t-d-1})}{1 - r_{2}}$$

$$= \frac{-\xi\lambda_{1}(1 - (1 - \lambda_{1})^{t-d-1})}{\lambda_{1}} + \frac{\xi\lambda_{2}(1 - (1 - \lambda_{2})^{t-d-1})}{\lambda_{2}}$$

$$= \xi \left[ (1 - \lambda_{1})^{t-d-1} - (1 - \lambda_{2})^{t-d-1} \right]$$
(A.3)

If  $\lambda_1 < \lambda_2$ , then  $B_t(\xi, \lambda_1, d) < B_t(\xi, \lambda_2, d)$ . Since we also have  $B_t(\xi, \lambda, d) > 0$ , it follows that  $SSE(\xi, \lambda_1, d) < SSE(\xi, \lambda_2, d)$ .

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Ming-Feng Wu received the B.S. degree from National Taiwan University of Science and Technology, Taipei, Taiwan, R.O.C., in 2000, and the M.S. degree from National Tsing-Hua University, Hsin Chu, Taiwan, in 2002, both in chemical engineering. He is currently working toward the Ph.D. degree at National Tsing-Hua University.

He is a specialist in multivariant statistical analysis. His work is focused on the derivation of run-to-run control and virtual metrology for chemical–vapor deposition processes.



**Chien-Hua Lin** received the B.S. degree in mathematics from Cheng-kung University, the M.S. degree in applied mathematics from Cheng-kung University, and the Ph.D. degree in statistics from Tsing-Hua University, Taiwan., R.O.C.

Currently, he does Postdoctoral Research with Prof. Tseng in the Institute of Statistics at Tsing-Hua University, Taiwan. His research interests include advanced process control and process monitoring.



**David Shan-Hill Wong** received the B.S. degree from the California Institute of Technology, Pasadena, in 1978, and the Ph.D. degree from the University of Delaware, Newark, in 1982, both in chemical engineering.

He has been a faculty member of the Chemical Engineering Department, National Tsing-Hua University, Hsinchu, Taiwan, R.O.C., since 1983. His research interests include the area of process system engineering and control of energy efficient separation processes and advanced process control in batch-

based manufacturing. He is actively involved in consulting and collaborative research with chemical process and semiconductor manufacturing industries.



Shi-Shang Jang received the M.S. degree from National Taiwan University, Taipei, Taiwan, R.O.C., in 1980, and the Ph.D. degree from Washington University, St. Louis, MO, in 1986, both in chemical engineering.

Since 1992, he has been a Professor in the Chemical Engineering Department, National Tsing-Hua University, Hsinchu, Taiwan. He was the Chairman of the department from 2000 to 2004. He is particularly interested in run-to-run control theory and practice.



**Sheng-Tsaing Tseng** (M'92) received the B. S. degree in business mathematics from Soochow University, the M.S. degree in applied mathematics from Tsing-Hua University, and the Ph.D. degree in management science from Tamkang University, Taiwan.

Currently, he is a Professor in the Institute of Statistics at Tsing-Hua University, Taiwan. His research interests include quality and productivity improvement, reliability lifetime analysis, and statistical decision methodology. His articles have appeared in numerous technical journals, such as

Technometrics, Journal of Quality Technology, IEEE TRANSACTIONS ON SEMICONDUCTOR MANUFACTURING, IEEE TRANSACTIONS ON RELIABILITY, and *IIE Transactions*.

Dr. Tseng has received the outstanding research award three times from the National Science Council (NSC) of R.O.C., in 1993, 1999, and 2004. Currently, he serves as an Associate Editor of *Journal of Statistical Planning and Inference* and he is an elected member of ISI.