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Journal of Process Control 16 (2006) 431–443

JOURNAL OF PROCESS CONTROL

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Stability and performance analysis of mixed product run-to-run control

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Received 2 June 2005; received in revised form 20 September 2005; accepted 26 September 2005

Abstract

Run-to-run control has been widely used in batch manufacturing processes to reduce variations. However, in batch processes, many different products are fabricated on the same set of process tool with different recipes. Two intuitive ways of defining a control scheme for such a mixed production mode are (i) each run of different products is used to estimate a common tool disturbance parameter, i.e., a "tool-based" approach, (ii) only a single disturbance parameter that describe the combined effect of both tool and product is estimated by results of runs of a particular product on a specific tool, i.e., a "product-based" approach. In this study, a model two-product plant was developed to investigate the "tool-based" and "product-based" approaches. The closed-loop responses are derived analytically and control performances are evaluated. We found that a "tool-based" approach is unstable when the plant is non-stationary and the plant-model mismatches are different for different products. A "product-based" control is stable but its performance will be inferior to single product control when the drift is significant. While the controller for frequent products can be tuned in a similar manner as in single product control, a more active controller should be used for the infrequent products which experience a larger drift between runs. The results were substantiated for a larger system with multiple products, multiple plants and random production schedule.

Keywords: Run-to-run control; Mixed product system; "Tool-based" control; "Product-based" control; Exponential weighted moving average (EWMA)

1. Introduction

As pointed out by Cussler and Moggridge [1], there has been a gradual shift of focus in the chemical process industry away from commodities into chemical products, which are high value-added and produced in only small quantities. Typically, many different products are manufactured in batch operations using the same set of generic equipments but different recipes. Suborn et al. [2] summarized control activities for such a production system into four categories: (i) logic control to follow steps of a prescribed recipe, (ii) within batch feedback control of process variables so that the batch history can follow a prescribed trajectory, (iii) run-to-run (RtR) control in which trends in product qualities such as shifts, drifts and patterned variations are eliminated by small recipe adjustments, and (iv) production management which includes diagnostics of the current status of the plant, scheduling production according to current resources, etc.

RtR control is an integrated form of statistical quality and engineering feed-back control. In the last decade, run-to-run (RtR) control has been widely applied in the semi-conductor manufacturing industry [3–5]. Active research in this area has been summarized by many authors in books and review articles. Topics investigated include the development of more sophisticated generic control algorithms [6–8] as well as control practices catered for

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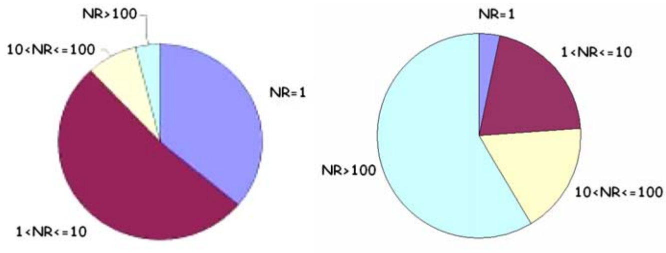
0959-1524/$ - see front matter
doi:10.1016/j.jprocont.2005.09.005



(a) Product Distribution (b) Run distribution

Fig. 1. Typical product (a) and run (b) distribution in a foundry.

specific operations such as chemical mechanical polishing and photolithography [9,10]. Most of these researches have been based on the assumption that there is only a single product in the manufacturing line. This is, however, far from realistic. Statistics of an unnamed operation during a certain period in a manufacturing line of a Taiwanese foundry are shown in Fig. 1. More than 70% of the products were produced for less than 10 runs (Fig. 1a). Only a few frequently produced products occupy more than 50% of the production runs. The rest of the operation are fabricating a large variety of products, each of which is produced only occasionally (Fig. 1b). Such a "high-mix" production mode leads to degradation in capacity as well as process capability.

Only a few studies have addressed the RtR control of a mixed product process plant. Patel et al. [11] proposed a control scheme that compensates tool induced, product induced and incoming disturbances for chemical mechanical polishing. In this method, a product dependence parameter known as sheet film equivalent was introduced for different products to estimate a unified removal rate for the tool. Edgar et al. [12] reviewed the problems of mixed product run-to-run control in a high-mix fab. One of the solutions suggested for the overlay process, in which one pattern layer is aligned on top of another named just-in-time adaptive disturbance estimation (JADE), is to break down the disturbances into contributions of the current product, the current tool and the previous reference recticle and reference tool using production data. Toprac and Wang [13] found that this approach is superior to a streamline approach in which control action is determined by previous runs with the same tool combination.

In both of the above methods, several disturbance factors that cause variations of qualities are observed using experimental data. Some are related to the deterioration of the manufacturing equipment itself and are termed "tool disturbances". Others are related to variations in physical and chemical properties, design and specifications of the context being processed, they are termed "product disturbances". It should be noted that in run-to-run control, a static gain model is used for the process input–output relationship. The disturbance to this model is a lumped factor of many causes that cannot be accounted for and measured. Such a simplistic input–output process model is bound to have uncertainty. Our ability to identify disturbance using historical operating data and a simplistic process model will be severely limited. The effect of model error on the stability of single product run-to-run control has been well documented [4,14–17]. Yet the effect of model uncertainty on the stability of mixed run control has never been investigated. In this work, a single- tool and two-product plant was developed to investigate two different intuitive control approaches, "tool-based" and "product-based". The closed-loop responses are derived analytically and control performances are evaluated. In the next section, the model of the plant and disturbances and the two control strategies will be introduced. In Section 3, we shall show that a "tool-based" approach is unstable if there are model error and non-stationary disturbances. In Section 4, the performance of "product-based" control, will be analyzed and benchmarked against a single-product plant. Guidelines for tuning such a control scheme will be examined. In the concluding section, a summary of the findings will be given. Details of mathematical derivations are given in appendices.

2. System model

2.1. Multi-product plant

Consider a simple case that two products are manufactured on a single tool (Fig. 2). The production schedule consists of cycles of i runs, in which j runs are used to produce Product 1 and $(i-j)$ runs are used to produce Product 2. j is called the campaign length of Product 1 and $i-j$ is defined as the break length for Product 1. Assume that the input–output relationships for the two products on the given tool are linear with different intercept α_1, α_2 and slopes β_1, β_2 and share the same tool disturbance η:

$$Y_{it+n} = \begin{cases} \alpha_1 + \beta_1 X_{it+n} + \eta_{it+n}, & 1 \leqslant n \leqslant j, \\ \alpha_2 + \beta_2 X_{it+n} + \eta_{it+n}, & j+1 \leqslant n \leqslant i, \end{cases} \tag{1}$$

where t is number of cycles, Y_{it+n} $(n = 1,2,\ldots,j)$, Y_{it+n} $(n = j+1,\ j+2,\ldots,i)$ are the outputs of Products 1

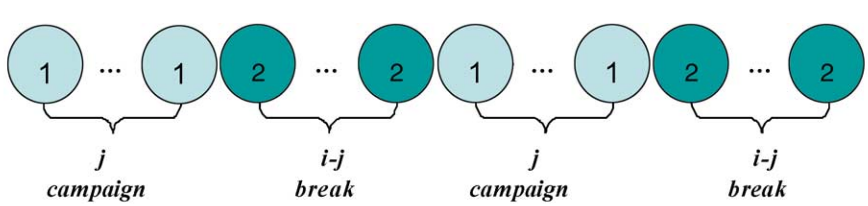


Fig. 2. Illustrative example of a tool manufacturing two products.

and 2, and X_{it+n} $(n = 1, 2, \ldots, i)$ are the control actions at run $it + n$. To model the change in tool condition, an IMA(1,1) disturbance with drift is used:

$$\eta_k - \eta_{k-1} = \delta + \varepsilon_k - \theta\varepsilon_{k-1}, \tag{2}$$

where δ is a deterministic drift and $\varepsilon_k \in N(0, \sigma^2)$ is a Gaussian distributed random noise with zero mean and variance σ^2.

2.2. Tool based EWMA control

In a "tool-based" control, a single "tool-noise" $\hat{\eta}_{it+n}$ $(n = 1, 2, \ldots, i)$ is estimated from input–output data and two linear models for the production of the two products

$$\hat{\eta}_{it+n} = \begin{cases} Y_{it+n} - (a_1 + b_1 X_{it+n}), & 1 \leqslant n \leqslant j, \\ Y_{it+n} - (a_2 + b_2 X_{it+n}), & j+1 \leqslant n \leqslant i. \end{cases} \tag{3}$$

a_1, a_2, b_1 and b_2 are fixed model parameters. This observed "tool noise" can be filtered

$$\tilde{\eta}_{it+n} = \lambda\hat{\eta}_{it+n} + (1 - \lambda)\tilde{\eta}_{it+n-1}, \tag{4}$$

where $0 \leqslant \lambda \leqslant 1$ is the discount factor of the exponential weighted moving average (EWMA) algorithm. Assuming the targets of the outputs are zero, the deadbeat control actions are

$$X_{it+n} = \begin{cases} \dfrac{0 - a_1 - \tilde{\eta}_{it+n-1}}{b_1}, & 1 \leqslant n \leqslant j, \\ \dfrac{0 - a_2 - \tilde{\eta}_{it+n-1}}{b_2}, & j+1 \leqslant n \leqslant i. \end{cases} \tag{5}$$

2.3. Product based control

In "product-based" control, the EWMA filter action is performed with respect to the last run on which the same product is processed instead of the previous run in which a different product may have been processed. Hence, for Product 1, the filtered plant noise can be expressed as

$$\tilde{\eta}_{it+n} = \begin{cases} \lambda(Y_{it+1} - b_1 X_{it+1} - a_1) + (1 - \lambda)\hat{\eta}_{i(t-1)+j}, \\ \quad n = 1, \\ \lambda(Y_{it+n} - b_1 X_{it+n} - a_1) + (1 - \lambda)\hat{\eta}_{it+n-1}, \\ \quad n = 2, \ldots, j. \end{cases} \tag{6}$$

The deadbeat control actions for Product 1 can be formulated as

$$X_{it+n} = \begin{cases} \dfrac{0 - a_1 - \tilde{\eta}_{(i-1)t+j}}{b_1}, & n = 1, \\ \dfrac{0 - a_1 - \tilde{\eta}_{it+n-1}}{b_1}, & n = 2, 3, \ldots, j. \end{cases} \tag{7}$$

It should be pointed out that in this control scheme, the quality of Product 1 becomes independent of what is produced in other runs $it + n, j + 1 \leqslant n \leqslant i$, i.e., whether a single product or a variety of products are being produced in these runs.

3. Stability of tool based control

In controller design, the stability of a control scheme is of primary importance. An unstable control scheme should not be implemented. In this section, we shall examine the influence of model error and the nature of disturbances on the stability of the "tool-based" scheme. First, the close-loop response of the scheme at the start of each production cycle is obtained.

Lemma 1. *The output at the first run of the $t + 1$ production cycle is given by*

$$\begin{aligned} Y_{it+i+1} = \frac{p}{1-q} &+ \sum_{k=0}^{t-1} q^k \Bigg[q_2^{i-j} \sum_{m=0}^{j-2} q_1^{m+1} (\eta_{it+j-m-ki} - \eta_{it+j-m-1-ki}) \\ &+ \xi_2^{-1}\xi_1 q_2^{i-j} (\eta_{it+j+1-ki} - \xi_1^{-1}\xi_2\eta_{it+j-ki}) \\ &+ \xi_2^{-1}\xi_1 \sum_{m=0}^{i-j-2} q_2^{m+1} (\eta_{it+i-m-ki} - \eta_{it+i-m-1-ki}) \\ &+ \left(\eta_{it+i+1-ki} - \xi_2^{-1}\xi_1\eta_{it+i-ki}\right) \Bigg] \end{aligned} \tag{8}$$

with $\xi_n = \beta_n/b_n$, $n = 1, 2$ being the modelling errors in process gain, and

$$\begin{aligned} P = {} & (1 - \lambda\xi_2)^{i-j}[\xi_2^{-1}(\xi_1\alpha_2 - \xi_2\alpha_1) + \xi_1(a_1 - a_2)] \\ & + \xi_2^{-1}(\xi_2\alpha_1 - \xi_1\alpha_2) - \xi_1(a_1 - a_2), \end{aligned} \tag{9}$$

$$q_1 = 1 - \lambda\xi_1, \quad q_2 = 1 - \lambda\xi_2, \quad q = q_2^{i-j} q_1^j. \tag{10}$$

Proof. See Appendix A for algebraic details. □

Given the closed-loop response, its asymptotic behavior can be obtained and the stability of the controller can be determined.

Theorem 1. *The process is unstable by "tool-based" control if*

(1) $q \geqslant 1$;
(2) $0 \leqslant q < 1$ *and the disturbance is non-stationary, i.e., contains a deterministic drift* ($\delta \neq 0$) *or a stochastic integrating part* ($\theta \neq 1$);
(3) $0 \leqslant q < 1$ *and there is a difference in error of the process gain estimates between products* ($\xi_1 \neq \xi_2$).

Proof. If $q \geqslant 1$, the output Y_{it+i+1} is unstable according to (8).

If $0 \leqslant q < 1$, for an IMA(1,1) noise,

$$\begin{aligned} \eta_n &= \eta_{n-1} + \delta + \varepsilon_n - \theta\varepsilon_{n-1} = \cdots \\ &= n\delta + \varepsilon_n + (1 - \theta)\sum_{j=1}^{n-1}\varepsilon_j, \end{aligned} \tag{11}$$

the term $q^k(\eta_{it+j+1-ki}-\xi_1^{-1}\xi_2\eta_{it+j-ki})$ in Eq. (8) can be computed as

$$\begin{aligned} q^k(\eta_{it+j+1-ki}-\xi_1^{-1}\xi_2\eta_{it+j-ki}) = q^k\xi_1^{-1}\{&[(it+j-ki)(\xi_1-\xi_2)\\ &+\xi_1]\delta+(1-\theta)(\xi_1-\xi_2)\\ &\times(\varepsilon_1+\varepsilon_2+\cdots+\varepsilon_{it+j-ki-1})\\ &+[(1-\theta)\xi_1-\xi_2]\varepsilon_{it+j-ki}\\ &+\xi_1\varepsilon_{it+j-ki+1}\}. \end{aligned} \quad (12)$$

As t approaches infinity, if $\xi_1\neq\xi_2$ and $\delta\neq 0$, then $q^k\xi_1^{-1}[(it+j-ki)(\xi_1-\xi_2)]\delta$ will diverge to infinity for the term $k=0$. Y_{it+i+1} also diverges and the process becomes unstable. Similarly, if $\xi_1\neq\xi_2$, and $\theta\neq 1$, the term $(1-\theta)(\xi_1-\xi_2)(\varepsilon_1+\varepsilon_2+\cdots+\varepsilon_{it+j-ki-1})$ becomes an infinite sum of random numbers. Even though the mean value $\langle Y_{it+i+1}\rangle$ is finite, the variance of $\langle(Y_{it+i+1}-\langle Y_{it+i+1}\rangle)^2\rangle$ will diverge. □

The performance of a controller can be evaluated by the asymptotic mean square error (AMSE).

$$\mathrm{AMSE}(Y_t)=\lim_{t\to\infty}\lfloor Y_t^2\rfloor. \quad (13)$$

AMSE can be determined analytically if the disturbance is a white noise or there is no discrepancy in error of process gains estimates between products.

Corollary 1. *If $0\leqslant q<1$ and the disturbance is a white noise, then the "tool-based" EWMA control is stable and the asymptotic mean square error (AMSE) of product n is given by*

$$\begin{aligned} \mathrm{AMSE}(Y_{it+i+n}) = \left(\frac{pq_1^{n-1}}{1-q}\right)^2+\sigma^2\Bigg[&\gamma_3\left(1+(q_1-1)^2\frac{1-q_1^{2n-4}}{1-q_1^2}\right.\\ &\left.+q_1^{2n-4}-2q_1^{2n-3}\right)+\frac{q_1^{2n-2}S_{tw}-2q_2^{i-j}q_1^{2n+j-3}q}{1-q^2}\Bigg],\\ n=1,\ldots,j \end{aligned} \quad (14)$$

with

$$\begin{aligned} S_{tw} = q_2^{2i-2j}&\left(1+2\gamma_1\frac{q_1(q_1^{2j-1}-1)}{1+q_1}\right)+\xi_2^{-2}\xi_1^2\\ &\times\left(1+q_2^{2i-2j}-(1-\gamma_2)q_2^{i-j}-2\gamma_2\frac{q_2(q_2^{2i-2j-1}+1)}{1+q_2}\right)+1, \end{aligned} \quad (15)$$

$$\gamma_1=\begin{cases}1, & j\geqslant 2,\\ 0, & j<2,\end{cases}\quad \gamma_2=\begin{cases}1, & i-j\geqslant 2,\\ 0, & i-j<2,\end{cases}\quad \gamma_3=\begin{cases}1, & n\geqslant 2,\\ 0, & n<2.\end{cases} \quad (16)$$

Proof. See Appendix B for the procedure. □

Corollary 2. *If $0\leqslant q<1$ and there is no discrepancy in error of process gain estimates between products, i.e., $\xi_1=\xi_2$, then the "tool-based" EWMA control is stable and the AMSE of Product 1 is given by*

$$\begin{aligned} \mathrm{AMSE}(Y_{it+i+n}) = &\left[\frac{p'q_1^{n-1}}{1-q'}+\left(\frac{1-q_1^{n-1}}{1-q_1}+\frac{q_1^{n-1}S_{td}}{1-q'}\right)\delta\right]^2\\ &+\gamma_3\left[1+(q_1-\theta)^2\frac{1-q_1^{2n-4}}{1-q_1^2}+q_1^{2n-4}\theta^2\right.\\ &\left.-2q_1^{2n-3}\theta\right]\sigma^2+\frac{q_1^{2n-2}S_{ti}-2q_1^{i+2n-3}q'\theta}{1-q'^2}\sigma^2,\\ n=1,\ldots,j \end{aligned} \quad (17)$$

with

$$\begin{aligned} p' = &(1-\lambda\xi_1)^{i-j}[(\alpha_2-\alpha_1)+\xi_1(a_1-a_2)]\\ &+(\alpha_1-\alpha_2)-\xi_1(a_1-a_2), \end{aligned} \quad (18)$$

$$q'=q_1^i, \quad (19)$$

$$S_{td}=\frac{1-q'}{1-q_1}, \quad (20)$$

$$\begin{aligned} S_{ts} = q_1^{2i-2j}&\left[\gamma_1 q_1^{2j-2}\theta^2+\gamma_1\frac{(q_1-\theta)^2(q_1^2-q_1^{2j-2})}{1-q_1^2}\right.\\ &\left.+(\gamma_1 q_1-\theta)^2+q_1^{-2}(q_1-\gamma_2\theta)^2\right]\\ &+\gamma_2\frac{(q_1-\theta)^2(q_1^2-q_1^{2i-2j-2})}{1-q_1^2}+(\gamma_2 q_1-\theta)^2+1. \end{aligned} \quad (21)$$

Proof. See Appendix C for the procedure. □

Since the plant-model mismatches for different products are unlikely to be the same in an actual plant, a "tool-based" control can be applied only when if the process disturbance contains neither a deterministic drift nor any integrating stochastic components. Fig. 3 illustrates the divergence scenarios of a "tool-based" multi-product EWMA. Fig. 3(a) shows that a deterministic drift with white noise results in rapidly increasing oscillatory responses. Fig. 3(b) shows that even with no drift, the diverging walk nature of a random walk noise cannot be eliminated. On the other hand, when the model errors of the two products are the same, the process is stable even when there are both deterministic drift and non-stationary stochastic noise, as illustrated in Fig. 3(c). When the two products have the same model errors, a "tool-based" control is equivalent to a single product control. A "tool-based" control remained stable also when the process disturbance is a stationary white noise (Fig. 3(d)). Yet if the process disturbance contains stationary white noise, there is no need for run-to-run control at all!

It is well known that for a single product plant, EWMA run-to-run control will eliminate constant offsets and reduce a drift into constant bias. Here, we showed that the ability of reducing drift to bias will be lost when a "tool-based" EWMA is applied to a multi-product tool when there is discrepancy in model errors between products.

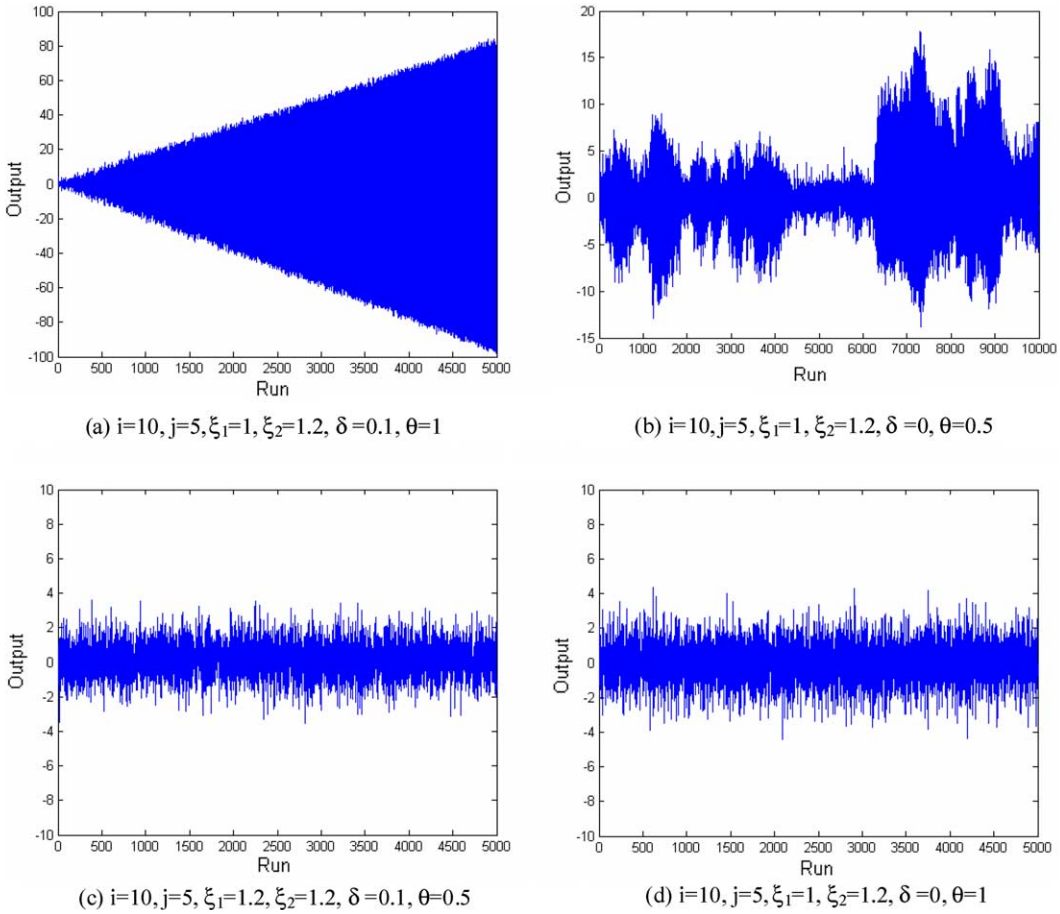


Fig. 3. Divergence of a "tool-based" EWMA control.

4. Performance of product based control

In order to evaluate the asymptotic performance of the product based control, the closed loop error and its square at the nth position in the tth production cycle is obtained first.

Lemma 2. *The following expressions for the closed loop response Y_{it+n} and its square at the nth position $1 \leqslant n < j$ in the tth production cycle using "product-based" EWMA control can be obtained*:

$$Y_{it+n} = \sum_{k=0}^{k_0-1} q_1^k (\eta_{f(k)} - \eta_{f(k)-m'}), \tag{22}$$

$$Y_{it+n}^2 = \sum_{k'=0}^{k_0-1} \sum_{k=0}^{k_0-1} q_1^{k+k'} (\eta_{f(k)} - \eta_{f(k)-m})(\eta_{f(k)} - \eta_{f(k)-m'}) \tag{23}$$

with

$$k_0 = jt + n, \tag{24}$$

$$f(k) = it + n - \left[\frac{k}{j}\right]_{\text{Int}} i - \left[\frac{k}{j}\right]_{\text{Rem}}, \tag{25}$$

$$m, m' = \begin{cases} i - j + 1, & \left[\frac{f(k)}{j}\right]_{\text{Rem}} = 1, \\ 1, & \left[\frac{f(k)}{j}\right]_{\text{Rem}} = 0, 2, \ldots, j-1 \end{cases} \tag{26}$$

with $[\ ]_{\text{Int}}$ and $[\ ]_{\text{Rem}}$ denoting integer quotient and remainder of division.

Proof. See Appendix D for details. □

By substituting the expression (23) for an IMA(1,1) noise with drift, the asymptotic mean square error is calculated:

Lemma 3. *Given $0 \leqslant q_1 < 1$ and a process disturbance that is IMA(1,1) with drift, the AMSE for the response at the nth position of the production cycle using a "product-based" EWMA control is given by*

$$\text{AMSE}(Y_n) = S_{pd}(n)\delta^2 + \frac{S_{ps}(n) - 2q_1^{2j-1}\theta}{1 - q_1^{2j}}\sigma^2, \quad 1 \leqslant n \leqslant j, \tag{27}$$

$$S_{pd}(n) = \left[\frac{q_1^{n-1}(i-j)}{1-q_1^j} + \frac{1}{1-q_1}\right]^2, \tag{28}$$

$$S_{ps}(n) = 1 + \frac{(q_1-\theta)^2(1-q_1^{2j-2})}{1-q_1^2} + q_1^{2j-2}\theta^2 + q_1^{2n-2}(1-\theta)^2(i-j). \tag{29}$$

Proof. See Appendix E for details. □

By taking the average of all the positions, the average AMSE of Product 1 can be obtained.

Theorem 2. *If* $0 \leqslant q_1 < 1$, *the average AMSE over all different positions of the campaign cycle of Product* 1 *is given by*

$$\mathrm{AMSE}_1(i,j) = \langle S_{pd}\rangle(i,j)\delta^2 + \frac{\langle S_{ps}\rangle(i,j) - 2q_1^{2j-1}\theta}{1-q_1^{2j}}\sigma^2, \tag{30}$$

$$\langle S_{pd}\rangle(i,j) = \left(\frac{1}{1-q_1}\right)^2 + \frac{(1-q_1^{2j})}{(1-q_1^j)^2(1-q_1^2)}\frac{(i-j)^2}{j} + \frac{2}{(1-q_1)^2}\frac{i-j}{j}, \tag{31}$$

$$\langle S_{ps}\rangle(i,j) = 1 + \frac{(q_1-\theta)^2(1-q_1^{2j-2})}{1-q_1^2} + q_1^{2j-2}\theta^2 + \frac{(1-q_1^{2j})}{1-q_1^2}\frac{i-j}{j}(1-\theta)^2. \tag{32}$$

Proof. See Appendix F for details. □

The argument (i,j) is used to denote that this is the closed-loop performance obtained for a particular schedule. Since a finite value for the AMSE is obtained, the control scheme is stable. It is necessary for us to benchmark its performance.

Corollary 3. *If* $0 \leqslant q_1 < 1$, *the AMSE for Product* 1 *will always be greater than the single product EWMA*:

$$\mathrm{AMSE}_1(i,j) \geqslant \mathrm{AMSE}_1(1,1).$$

Proof. See Appendix G for details. □

The above corollary shows that the "product-based" run-to-run control performance of a mixed product tool will be inferior to that of a single product tool. Let us examine the magnitude of this degradation for different situations. Consider a frequent product which comprises a substantial portion of the runs, e.g., $j/i = 50\%$, and an infrequent product which makes up a small fraction of the runs, e.g., $j/i = 10\%$. The error in model gains are assumed to be $\xi_1 = \xi_2 = 0.8$.

Fig. 4 compares the AMSEs obtained for the frequent product at different values of the tuning parameter λ and campaign lengths when there is a small deterministic drift $\delta = 0.01$ and a large drift $\delta = 0.1$. When there is a small drift, as in the case of single product, only a small λ is needed and the degradation in performance is minimal. The effect of campaign length on the AMSE is negligible, i.e., no advantage is gained by lumping similar products into long campaigns. When there is a significant deterministic drift, the optimal λ obtained is about 0.57 which is larger than the value of 0.39 obtained for single product control. The optimal AMSE obtained is about 1.50, which is about 16% degradation from the value of 1.29 for a single product plant. Again the effect of length of a continuous campaign is negligible.

Fig. 5 presents similar comparison for an infrequent product. The optimal λ obtained is about 0.4 even when the drift is very small. The optimal AMSE obtained is about 1.29, which is about 22% degradation from the value of 1.05 for a single product plant. When there is a large drift, a very large value of $\lambda \sim 1$ should be used. Even with such an active controller, the AMSE is still very high (3.2 for $i = 10$), and increases as the time elapsed between runs of the same product, i.e., the break length, increases.

5. Simulation study

If there are more than two products and one tool, or if the production schedule is random, the system becomes too complicated for rigorous analysis. A simulation example is provided to illustrate fee arguments presented above. Consider the following simulation example in which there are two tools and five products. The parameters of linear plant for each of the five products are $\bar{\alpha} = [1,2,3,2,2]$ and $\bar{\beta} = [1,2,3,1.5,2.5]$. The parameters of controller models

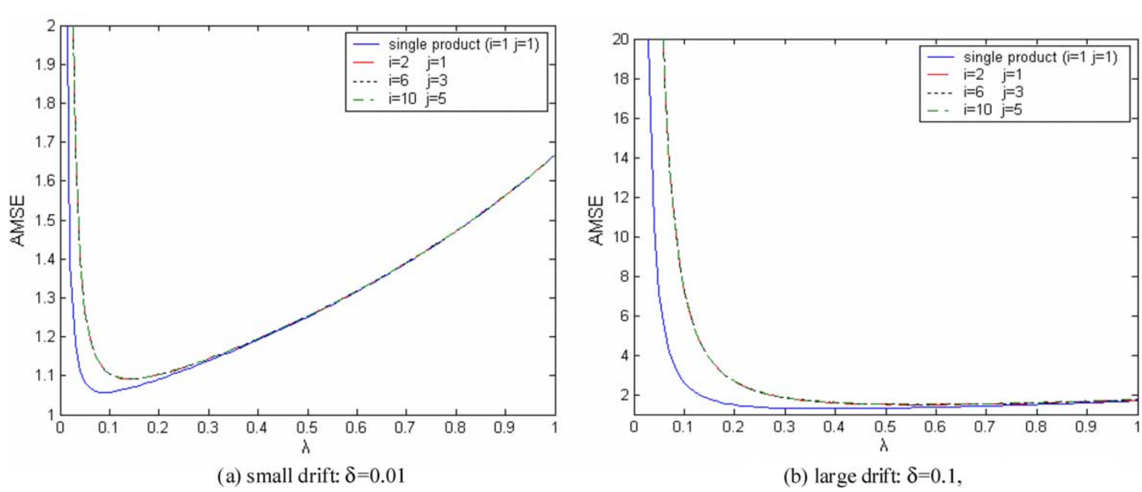
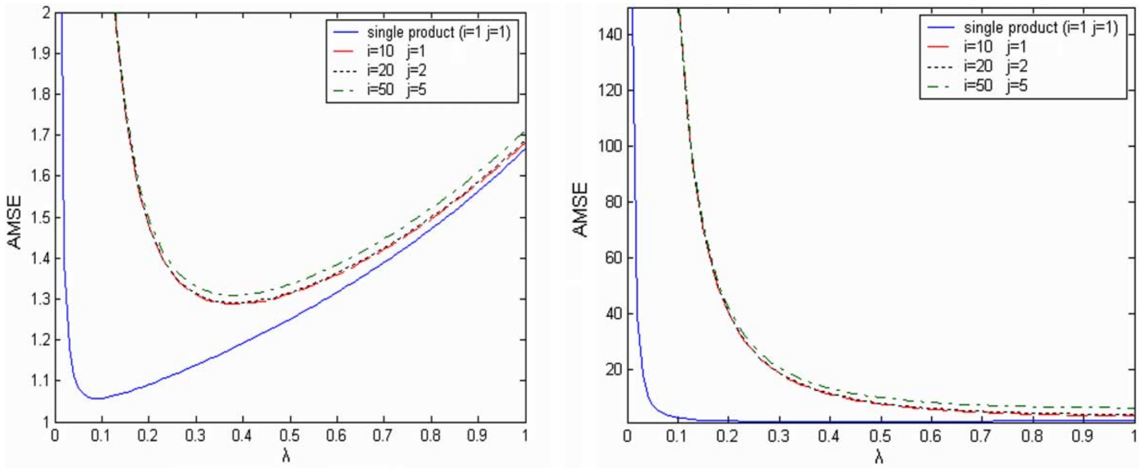


Fig. 4. Performance of a frequent product in a "product-based" control ($j/i = 0.5$, $\xi_1 = 0.8$, $\theta = 1$, $\sigma = 1$).



(a) small drift: δ=0.01 (b) large drift: δ=0.1,

Fig. 5. Performance of an infrequent product in a "product-based" control ($j/i = 0.1$, $\xi_1 = 0.8$, $\theta = 1$, $\sigma = 1$).

are $\bar{a} = [1, 2, 3, 1.5, 2.5]$ and $\bar{b} = [0.8, 1.4, 2.4, 1.5, 3.0]$. The model mismatch in process gains are $\bar{\xi} = [0.8, 0.7, 0.8, 1, 1.2]$. The disturbances of two tools are both IMA(1, 1) noise with drifts. The drift and moving averaging parameters are $\bar{\delta} = [0.1, 0.2]$ and $\bar{\theta} = [0.3, 0.7]$, while the white noises are sequences with zero mean and unit variance $\varepsilon = N(0, 1)$. Fig. 6 presents the output of tools 1 and 2 based on a "product-based" single EWMA control with $\lambda = 0.4$. It shows that "product-based" approach is able to eliminate the drift, although substantial offsets and large variations were found. Fig. 7 shows that a "tool-based" control approach is unable to eliminate the drift and the system ran away rapidly. The closed loop response soon diverges rapidly. The method JADE [12,13] is also included for comparison in Fig. 8. A brief description of JADE is included in Appendix H. In JADE, individual contributions to bias parameters by different products and tools are estimated using a weighted recursive least square estimation. For details of the JADE method, readers are referred to Firth's thesis [18]. Fig. 8 shows that JADE also fail to eliminate the drift However, the divergence is much less severe than "tool-based" control approach.

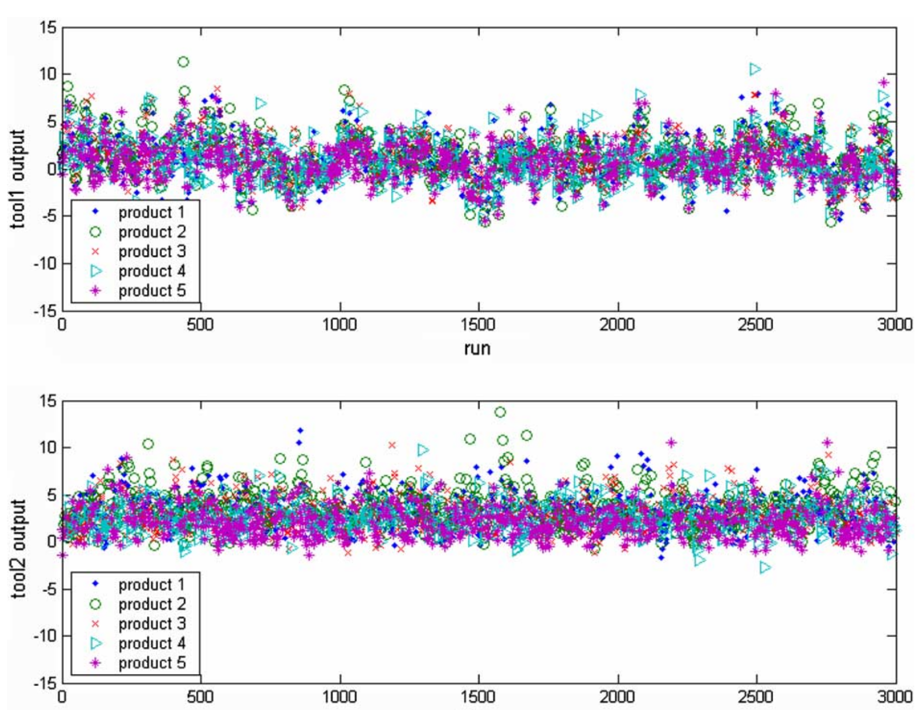


Fig. 6. The system output using "product-based" control approach with $\lambda = 0.4$.

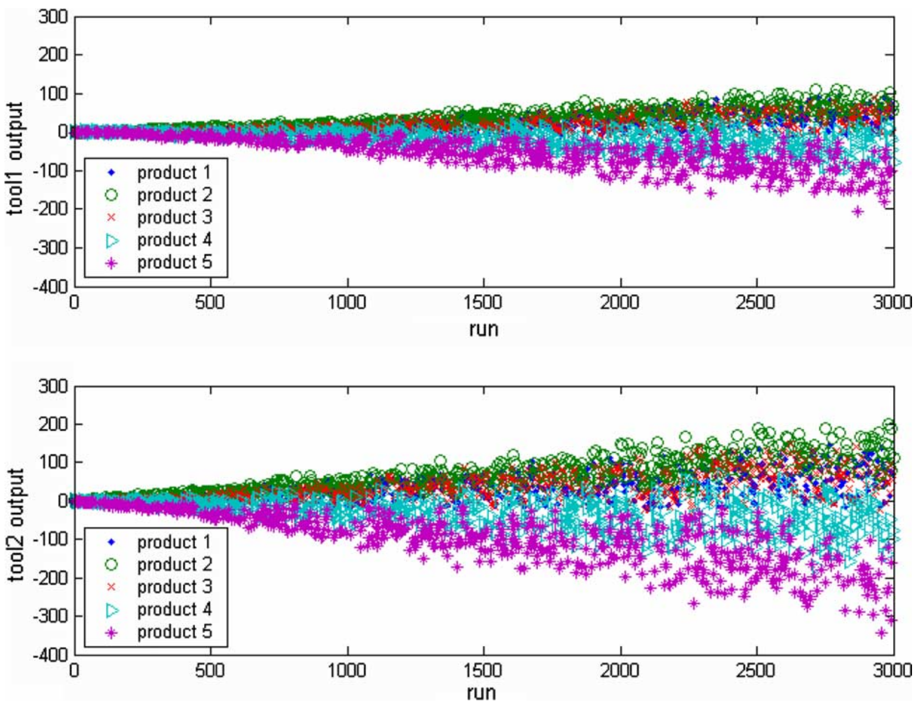


Fig. 7. The system output using a "tool-based" control approach with $\lambda = 0.4$.

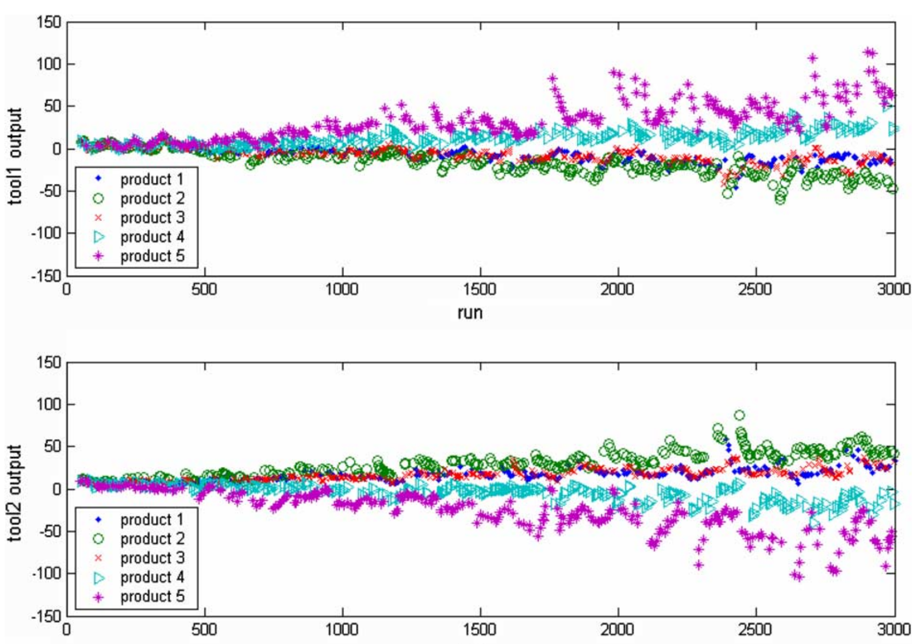


Fig. 8. The system output using the JADE approach with $\lambda = 0.4$.

6. Conclusions

In this work, a rigorous analysis of two different run-to-run control strategies in a model two-product plant is performed. The above plant is idealistic compared to actual operations. Hence, any control practice that fails for such a plant is unlikely to be applicable for real-life operation. On the other hand, a control solution that is viable for this model plant must be subjected to further scrutiny for on-line applications. We found that a "tool-based" approach is unstable when the plant is non-stationary and the plant-model mismatches are different for different products. Therefore, one must caution against putting too much confidence into separation of "product" and "tool" contributions to the disturbance. Such separation would work only if the process shows little drift and the process gain is well characterized. A "product-based" control is stable but its performance will be inferior to single product control when the drift is significant. A more active controller should be used for the infrequent products, which experience a larger drift between runs. Alternatively, it is advantageous to bunch the production of an infrequent product in a short period with longer campaign lengths. The conclusion that the tool-based approach becomes unstable when there are differences in model uncertainty between different products and tool drifts was found to be also valid for a larger system with two tools and five different products by simulation.

Acknowledgement

The authors thank the financial support for this work from National Science Council, Taiwan, through the grant NSC-92-2214-E007-008.

Appendix A

Combining (5) and (1), we get

$$Y_{it+j} = (1-\lambda\xi_1)^{j-1} Y_{it+1} + \sum_{m=0}^{j-2} (1-\lambda\xi_1)^m (\eta_{it+j-m} - \eta_{it+j-m-1}), \tag{A.1}$$

$$Y_{it+j+1} = \xi_1^{-1}\xi_2(1-\lambda\xi_1)Y_{it+j} + \xi_1^{-1}(\xi_1\alpha_2 - \xi_2\alpha_1 + \xi_1\xi_2(a_1-a_2) + \xi_1\eta_{it+j+1} - \xi_2\eta_{it+j}), \tag{A.2}$$

$$Y_{it+i} = (1-\lambda\xi_2)^{i-j-1} Y_{it+j+1} + \sum_{m=0}^{i-j-2} (1-\lambda\xi_2)^m (\eta_{it+i-m} - \eta_{it+i-m-1}), \tag{A.3}$$

$$Y_{it+i+1} = \xi_2^{-1}\xi_1(1-\lambda\xi_2)Y_{it+i} + \xi_2^{-1}(\xi_2\alpha_1 - \xi_1\alpha_2 + \xi_1\xi_2(a_2-a_1) + \xi_2\eta_{it+i+1} - \xi_1\eta_{it+1}). \tag{A.4}$$

Substituting (A.1)–(A.3) into (A.4), we get

$$\begin{aligned} Y_{it+i+1} &= (1-\lambda\xi_2)^{i-j}(1-\lambda\xi_1)^j Y_{it+1} + (1-\lambda\xi_2)^{i-j} \\ &\quad\times \sum_{m=0}^{j-2}(1-\lambda\xi_1)^{m+1}(\eta_{it+j-m} - \eta_{it+j-m-1}) \\ &\quad+ \xi_2^{-1}\xi_1(1-\lambda\xi_2)^{i-j}(\eta_{it+j+1} - \xi_1^{-1}\xi_2\eta_{it+j}) \\ &\quad+ \xi_2^{-1}\xi_1 \sum_{m=0}^{i-j-2}(1-\lambda\xi_2)^{m+1}(\eta_{it+i-m} - \eta_{it+i-m-1}) \\ &\quad+ (\eta_{it+i+1} - \xi_2^{-1}\xi_1\eta_{it+i}) + (1-\lambda\xi_2)^{i-j} \\ &\quad\times [\xi_2^{-1}(\xi_1\alpha_2 - \xi_2\alpha_1) + \xi_1(a_1-a_2)] \\ &\quad+ \xi_2^{-1}(\xi_2\alpha_1 - \xi_1\alpha_2) - \xi_1(a_1-a_2). \end{aligned} \tag{A.5}$$

Define

$$p = (1-\lambda\xi_2)^{i-j}[\xi_2^{-1}(\xi_1\alpha_2 - \xi_2\alpha_1) + \xi_1(a_1-a_2)] + \xi_2^{-1}(\xi_2\alpha_1 - \xi_1\alpha_2) - \xi_1(a_1-a_2), \tag{A.6}$$

$$q_1 = 1-\lambda\xi_1, \quad q_2 = 1-\lambda\xi_2, \quad q = q_2^{i-j}q_1^j, \tag{A.7}$$

we have the following recursive relation:

$$\begin{aligned} Y_{it+i+1} &= p + qY_{it+1} + q_2^{i-j}\sum_{m=0}^{j-2} q_1^{m+1}(\eta_{it+j-m} - \eta_{it+j-m-1}) \\ &\quad+ \xi_2^{-1}\xi_1 q_2^{i-j}(\eta_{it+j+1} - \xi_1^{-1}\xi_2\eta_{it+j}) \\ &\quad+ \xi_2^{-1}\xi_1\sum_{m=0}^{i-j-2} q_2^{m+1}(\eta_{it+i-m} - \eta_{it+i-m-1}) \\ &\quad+ (\eta_{it+i+1} - \xi_2^{-1}\xi_1\eta_{it+i}), \end{aligned} \tag{A.8}$$

which can be rewritten as

$$\begin{aligned} Y_{it+i+1} &= q^t Y_1 + \sum_{k=0}^{t-1} q^k p \\ &\quad+ \sum_{k=0}^{t-1} q^k \Bigg[q_2^{i-j}\sum_{m=0}^{j-2} q_1^{m+1}(\eta_{it+j-m-ki} - \eta_{it+j-m-1-ki}) \\ &\quad+ \xi_2^{-1}\xi_1 q_2^{i-j}(\eta_{it+j+1-ki} - \xi_1^{-1}\xi_2\eta_{it+j-ki}) \\ &\quad+ \xi_2^{-1}\xi_1\sum_{m=0}^{i-j-2} q_2^{m+1}(\eta_{it+i-m-ki} - \eta_{it+i-m-1-ki}) \\ &\quad+ (\eta_{it+1-ki} - \xi_2^{-1}\xi_1\eta_{it+i-ki})\Bigg]. \end{aligned} \tag{A.9}$$

Assuming that $0 \leqslant q < 1$ and $Y_1 = 0$, we get

$$\begin{aligned} Y_{it+i+1} &= \frac{p}{1-q} + \sum_{k=0}^{t-1} q^k \Bigg[q_2^{i-j}\sum_{m=0}^{j-2} q_1^{m+1}(\eta_{it+j-m-ki} - \eta_{it+j-m-1-ki}) \\ &\quad+ \xi_2^{-1}\xi_1 q_2^{i-j}(\eta_{it+j+1-ki} - \xi_1^{-1}\xi_2\eta_{it+j-ki}) \\ &\quad+ \xi_2^{-1}\xi_1\sum_{m=0}^{i-j-2} q_2^{m+1}(\eta_{it+i-m-ki} - \eta_{it+i-m-1-ki}) \\ &\quad+ (\eta_{it+i+1-ki} - \xi_2^{-1}\xi_1\eta_{it+i-ki})\Bigg]. \end{aligned} \tag{A.10}$$

Appendix B

If disturbance is stationary white noise $\eta_n = \varepsilon_n$, we have

$$\begin{aligned} Y_{it+i+n} &= \frac{pq_1^{n-1}}{1-q} + \gamma_3\varepsilon_{it+i+n-ki} \\ &\quad+ \sum_{m=0}^{n-3} q_1^m(q_1-1)\varepsilon_{it+i+n-1-m} - \gamma_3 q_1^{n-2}\varepsilon_{it+i+1} \\ &\quad+ q_1^{n-1}\sum_{k=0}^{t-1} q^k \Bigg[q_2^{i-j}\Bigg(-\gamma_1 q_1^{j-1}\varepsilon_{it+1-ki} \\ &\quad+ \sum_{m=1}^{j-2} q_1^m(q_1-1)\varepsilon_{it+j-m-ki}\Bigg) + q_2^{i-j}(\gamma_1 q_1 - 1)\varepsilon_{it+j-ki} \end{aligned}$$

$$+\xi_2^{-1}\xi_1 q_2^{i-j-1}(q_2-\gamma_2)\varepsilon_{it+j+1-ki} + \xi_2^{-1}\xi_1\sum_{m=1}^{i-j-2} q_2^m(q_2-1)\varepsilon_{it+i-m-ki} + \xi_2^{-1}\xi_1(\gamma_2 q_2-1)\varepsilon_{it+i-ki}+\varepsilon_{it+i+1-ki}\Bigg] \tag{B.1}$$

with

$$\gamma_1=\begin{cases}1, & j\geqslant 2,\\ 0, & j<2,\end{cases} \tag{B.2}$$

$$\gamma_2=\begin{cases}1, & i-j\geqslant 2,\\ 0, & i-j<2,\end{cases} \tag{B.3}$$

$$\gamma_3=\begin{cases}1, & n\geqslant 2,\\ 0, & n<2.\end{cases} \tag{B.4}$$

Taking the square of (B.1), we get

$$\begin{aligned}Y^2_{it+i+n} &= \left(\frac{pq_1^{n-1}}{1-q}\right)^2 + \gamma_3\left[1+(q_1-1)^2\frac{1-q_1^{2n-4}}{1-q_1^2}+q_1^{2n-4}-2q_1^{2n-3}\right]\sigma^2 \\ &+ q_1^{2n-2}\sum_{k=0}^{t-1}q^k\Bigg[q_2^{i-j}\Bigg(-\gamma_1 q_1^{j-1}\varepsilon_{it+1-ki} + \sum_{m=1}^{j-2}q_1^m(q_1-1)\varepsilon_{it+j-m-ki}\Bigg)+q_2^{i-j}(\gamma_1q_1-1)\varepsilon_{it+j-ki} \\ &+ \xi_2^{-1}\xi_1 q_2^{i-j-1}(q_2-\gamma_2)\varepsilon_{it+j+1-ki} + \xi_2^{-1}\xi_1\sum_{m=1}^{i-j-2}q_2^m(q_2-1)\varepsilon_{it+i-m-ki} + \xi_2^{-1}\xi_1(\gamma_2q_2-1)\varepsilon_{it+i-ki}+\varepsilon_{it+i+1-ki}\Bigg] \\ &\times\sum_{k=0}^{t-1}q^{k'}\Bigg[q_2^{i-j}\Bigg(-\gamma_1q_1^{j-1}\varepsilon_{it+1-k'i} + \sum_{m=1}^{j-2}q_1^m(q_1-1)\varepsilon_{it+j-m-k'i}\Bigg)+q_2^{i-j}(\gamma_1q_1-1)\varepsilon_{it+j-k'i} \\ &+ \xi_2^{-1}\xi_1q_2^{i-j-1}(q_2-\gamma_2)\varepsilon_{it+j+1-k'i} + \xi_2^{-1}\xi_1\sum_{m=1}^{i-j-2}q_2^m(q_2-1)\varepsilon_{it+i-m-k'i} + \xi_2^{-1}\xi_1(\gamma_2q_2-1)\varepsilon_{it+i-k'i}+\varepsilon_{it+i+1-k'i}\Bigg].\end{aligned} \tag{B.5}$$

Taking the overall average and the limit $t\to\infty$

$$\begin{aligned}&\Bigg\langle\Bigg[q_2^{i-j}\Bigg(-\gamma_1q_1^{j-1}\varepsilon_{it+1-ki}+\sum_{m=1}^{j-2}q_1^m(q_1-1)\varepsilon_{it+j-m-ki}\Bigg) + q_2^{i-j}(\gamma_1q_1-1)\varepsilon_{it+j-ki}+\xi_2^{-1}\xi_1q_2^{i-j-1}(q_2-\gamma_2)\varepsilon_{it+j+1-ki} \\ &+\xi_2^{-1}\xi_1\sum_{m=1}^{i-j-2}q_2^m(q_2-1)\varepsilon_{it+i-m-ki} + \xi_2^{-1}\xi_1(\gamma_2q_2-1)\varepsilon_{it+i-ki}+\varepsilon_{it+i+1-ki}\Bigg] \\ &\times\Bigg[q_2^{i-j}\Bigg(-\gamma_1q_1^{j-1}\varepsilon_{it+1-k'i}+\sum_{m=1}^{j-2}q_1^m(q_1-1)\varepsilon_{it+j-m-k'i}\Bigg) + q_2^{i-j}(\gamma_1q_1-1)\varepsilon_{it+j-k'i}+\xi_2^{-1}\xi_1q_2^{i-j-1}(q_2-\gamma_2)\varepsilon_{it+j+1-k'i} \\ &+\xi_2^{-1}\xi_1\sum_{m=1}^{i-j-2}q_2^m(q_2-1)\varepsilon_{it+i-m-k'i}+\xi_2^{-1}\xi_1(\gamma_2q_2-1)\varepsilon_{it+i-k'i} + \varepsilon_{it+i+1-k'i}\Bigg]\Bigg\rangle = \begin{cases}S_{tw}\sigma^2, & k=k',\\ -q_2^{i-j}q_1^{j-1}\sigma^2, & |k-k'|=1,\\ 0, & |k-k'|\geqslant 2,\end{cases}\end{aligned} \tag{B.6}$$

where

$$S_{tw} = q_2^{2i-2j}\left(1+2\gamma_1\frac{q_1(q_1^{2j-1}-1)}{1+q_1}\right) + \xi_2^{-2}\xi_1^2\left(1+q_2^{2i-2j}-(1-\gamma_2)q_2^{i-j}-2\gamma_2\frac{q_2(q_2^{2i-2j-1})}{1+q_2}\right)+1. \tag{B.7}$$

Therefore, given $0\leqslant q<1$,

$$\mathrm{AMSE}(Y_{it+i+n}) = \left(\frac{pq_1^{n-1}}{1-q}\right)^2 + \left[\gamma_3\left(1+(q_1-1)^2\frac{1-q_1^{2n-4}}{1-q_1^2} + q_1^{2n-4}-2q_1^{2n-3}\right)+\frac{q_1^{2n-2}S_{tw}-2q_2^{i-j}q_1^{2n+j-3}q}{1-q^2}\right]\sigma^2. \tag{B.8}$$

Appendix C

According to (8) and (11), Y_{it+i+1} is

$$\begin{aligned}Y_{it+i+1} &= \frac{p+\left(q_2^{i-j}\frac{1-q_1^j}{1-q_1}+\frac{1-q_2^{i-j}}{1-q_2}\right)\delta}{1-q} + \sum_{k=0}^{t-1}q^k\Bigg\{q_2^{i-j}\Bigg[-\gamma_1q_1^{i-j}\theta\varepsilon_{it+1-ki} + \sum_{m=1}^{j-2}q_1^m(q_1-\theta)\varepsilon_{it+j-m-ki}\Bigg] + q_2^{i-j}(\gamma_1q_1-\theta)\varepsilon_{it+j-ki} \\ &+ q_2^{i-j-1}(q_2-\gamma_2^\theta)\varepsilon_{it+j+1-ki}+\sum_{m=1}^{i-j-2}q_2^m(q_2-\theta)\varepsilon_{it+i-m-ki} + \Big(\gamma_2q_2-\theta)\varepsilon_{it+i-ki}+\varepsilon_{it+i+1-ki}\Bigg\},\end{aligned}$$

when $\xi_1 = \xi_2$, so

$$\begin{aligned}
&q_2^{i-j}\left[-\gamma_1 q_1^{j-1}\theta\varepsilon_{it+1-ki} + \sum_{m=1}^{j-2} q_1^m (q_1-\theta)\varepsilon_{it+j-m-ki}\right]\\
&\quad + q_2^{i-j}(\gamma_1 q_1 - \theta)\varepsilon_{it+j-ki} + q_2^{i-j-1}(q_2 - \gamma_2\theta)\varepsilon_{it+j+1-ki}\\
&\quad + \sum_{m-1}^{i-j-2} q_2^m (q_2-\theta)\varepsilon_{it+i-m-ki} + (\gamma_2 q_2 - \theta)\varepsilon_{it+i-ki} + \varepsilon_{it+i+1-ki}\\
&\quad = S_{td}\delta + q_1^{i-j}\left[-\gamma_1 q_1^{j-1}\theta\varepsilon_{it+1-kt} + \sum_{m=1}^{j-2} q_1^m (q_1-\theta)\varepsilon_{it+j-m-kt}\right]\\
&\quad + q_1^{i-j}(\gamma_1 q_1 - \theta)\varepsilon_{it+j-kt} + q_1^{i-j-1}(q_1 - \gamma_2\theta)\varepsilon_{it+j+1-kt}\\
&\quad + \sum_{m=1}^{i-j-2} q_1^m (q_1-\theta)\varepsilon_{it+i-m-kt} + (\gamma_2 q_1 - \theta)\varepsilon_{it+i-kt} + \varepsilon_{it+i+1-kt},
\end{aligned}$$

where $S_{\text{td}} = \frac{1-q_1^i}{1-q_1}$.

If $0 \leqslant q < 1$ and $1 \leqslant n \leqslant j$, we can get

$$\begin{aligned}
Y_{it+i+n} &= q_1^{n-1} Y_{it+i+1} + \sum_{m=0}^{n-2} q_1^m (\eta_{it+i+n-m} - \eta_{it+i+n-m-1})\\
&= \frac{p' q_1^{n-1}}{1-q'} + \left(\frac{1-q_1^{n-1}}{1-q_1} + \frac{q_1^{n-1} S_{td}}{1-q'}\right)\delta + \gamma_3\varepsilon_{it+i+n-ki}\\
&\quad + \sum_{m=0}^{n-3} q_1^m (q_1-\theta)\varepsilon_{it+i+n-1-m} - \gamma_3 q_1^{n-2}\theta\varepsilon_{it+i+1}\\
&\quad + q_1^{n-1}\sum_{k=0}^{t-1} q'^k \Bigg\{ q_1^{i-j}\Bigg[-\gamma_1 q_1^{j-1}\theta\varepsilon_{it+1-ki}\\
&\quad + \sum_{m-1}^{j-2} q_1^m (q_1-\theta)\varepsilon_{it+j-m-ki}\Bigg] + q_1^{i-j}(\gamma_1 q_1 - \theta)\varepsilon_{it+j-ki}\\
&\quad + q_1^{i-j-1}(q_1-\gamma_2\theta)\varepsilon_{it+j+1-ki} + \sum_{m=1}^{i-j-2} q_1^m (q_1-\theta)\varepsilon_{it+i-m-ki}\\
&\quad + (\gamma_2 q_1 - \theta)\varepsilon_{it+i-ki} + \varepsilon_{it+i+1-ki}\Bigg\},
\end{aligned}$$

where $p' = (1-\lambda\xi_1)^{i-j}[(\alpha_2-\alpha_1) + \xi_1(a_1-a_2)] + (\alpha_1-\alpha_2) -\xi_1(a_1-a_2)$, $q' = q_1^i$,

$$\begin{aligned}
Y_{it+i+n}^2 &= \left[\frac{p' q_1^{n-1}}{1-q'} + \left(\frac{1-q_1^{n-1}}{1-q_1} + \frac{q_1^{n-1} S_{td}}{1-q'}\right)\delta\right]^2\\
&\quad + \gamma_3\left[1 + (q_1-\theta)^2\frac{1-q_1^{2n-4}}{1-q_1^2} + q_1^{2n-4}\theta^2 - 2q_1^{2n-3}\theta\right]\sigma^2\\
&\quad + q_1^{2n-2}\sum_{k'=0}^{t-1}\sum_{k=0}^{t-1} q'^{k+k'}\Bigg\{ q_1^{i-j}\Bigg[-\gamma_1 q_1^{j-1}\theta\varepsilon_{it+1-ki}\\
&\quad + \sum_{m=1}^{j-2}(q_1-\theta)\varepsilon_{it+j-m-ki}\Bigg] + q_1^{i-j}(\gamma_1 q_1-\theta)\varepsilon_{it+j-ki}\\
&\quad + q_1^{i-j-1}(q_1-\gamma_2\theta)\varepsilon_{it+j+1-ki} + \sum_{m=1}^{i-j-2} q_1^m(q_1-\theta)\varepsilon_{it+i-m-ki}\\
&\quad + \Bigg(\gamma_2 q_1 - \theta)\varepsilon_{it+i-ki} + \varepsilon_{it+i+1-ki}\Bigg\}\\
&\quad \times \Bigg\{ q_1^{i-j}\Bigg[-\gamma_1 q_1^{j-1}\theta\varepsilon_{it+1-k'i} + \sum_{m=1}^{j-2} q_1^m(q_1-\theta)\varepsilon_{it+j-m-k'i}\Bigg]\\
&\quad + q_1^{i-j}(\gamma_1 q_1-\theta)\varepsilon_{it+j-k'i} + q_1^{i-j-1}(q_1-\gamma_2\theta)\varepsilon_{it+j+1-k'i}\\
&\quad + \sum_{m=1}^{i-j-2} q_1^m(q_1-\theta)\varepsilon_{it+i-m-k'i} + (\gamma_2 q_1-\theta)\varepsilon_{it+i-k'i}\\
&\quad + \varepsilon_{it+i+1-k'i}\Bigg\}.
\end{aligned}$$

Since

$$\begin{aligned}
&\Bigg\{ q_1^{i-j}\Bigg[-\gamma_1 q_1^{j-1}\theta\varepsilon_{it+1-ki} + \sum_{m=1}^{j-2} q_1^m(q_1-\theta)\varepsilon_{it+j-m-ki}\Bigg]\\
&\quad + q_1^{i-j}(\gamma_1 q_1-\theta)\varepsilon_{it+j-ki} + q_1^{i-j-1}(q_1-\gamma_2\theta)\varepsilon_{it+j+1-ki}\\
&\quad + \sum_{m=1}^{i-j-2} q_1^m(q_1-\theta)\varepsilon_{it+i-m-ki} + (\gamma_1 q_1-\theta)\varepsilon_{it+i-ki} + \varepsilon_{it+i+1-ki}\Bigg\}\\
&\quad \times \Bigg\{ q_1^{i-j}\Bigg[-\gamma_1 q_1^{j-1}\theta\varepsilon_{it+1-k'i} + \sum_{m=1}^{j-2} q_1^m(q_1-\theta)\varepsilon_{it+j-m-k'i}\Bigg]\\
&\quad + q_1^{i-j}(\gamma_1 q_1-\theta)\varepsilon_{it+j-k'i} + q_1^{i-j-1}(q_1-\gamma_2\theta)\varepsilon_{it+j+1-k'i}\\
&\quad + \sum_{m=1}^{i-j-2} q_1^m(q_1-\theta)\varepsilon_{it+i-m-k'i} + (\gamma_2 q_1-\theta)\varepsilon_{it+i-k'i} + \varepsilon_{it+i+1-k'i}\Bigg\}\\
&\quad = \begin{cases} S_{ti}\sigma^2, & k = k',\\ -q_1^{i-1}\theta\sigma^2, & |k-k'| = 1,\\ 0, & |k-k'| \geqslant 2, \end{cases}
\end{aligned}$$

where

$$\begin{aligned}
S_{ti} &= q_1^{2i-2j}\Bigg(\gamma_1 q_1^{2j-2}\theta^2 + \gamma_1\frac{(q_1-\theta)^2(q_1^2-q_1^{2j-2})}{1-q_1^2} + (\gamma_1 q_1-\theta)^2\\
&\quad + q_1^{-2}(q_1-\gamma_2\theta)^2\Bigg) + \gamma_2\frac{(q_1-\theta)^2(q_1^2-q_1^{2i-2j-2})}{1-q_1^2}\\
&\quad + (\gamma_2 q_1-\theta)^2 + 1.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\text{AMSE}(Y_{it+i+n}) &= \left(\frac{p' q_1^{n-1}}{1-q'} + \left(\frac{1-q_1^{n-1}}{1-q_1} + \frac{q_1^{n-1} S_{td}}{1-q'}\right)\delta\right)^2\\
&\quad + \gamma_3\Bigg[1 + (q_1-\theta)^2\frac{1-q_1^{2n-4}}{1-q_1^2} + q_1^{2n-4}\theta^2\\
&\quad - 2q_1^{2n-3}\theta\Bigg]\sigma^2 + q_1^{2n-2}\sum_{k=0}^{t-1} q'2kS_{ti}\sigma^2\\
&\quad - 2q_1^{2n-2}\sum_{k=0}^{t-2} q'2k + 1(q_2^{i-j} q_1^{j-1}\theta\sigma^2)\\
&= \left(\frac{p' q_1^{n-1}}{1-q'} + \left(\frac{1-q_1^{n-1}}{1-q_1} + \frac{q_1^{n-1} S_{td}}{1-q'}\right)\delta\right)^2\\
&\quad + \gamma_3\Bigg[1 + (q_1-\theta)^2\frac{1-q_1^{2n-4}}{1-q_1^2} + q_1^{2n-4}\theta^2\\
&\quad - 2q_1^{2n-3}\theta\Bigg]\sigma^2 + \frac{q_1^{2n-2}S_{ti} - 2q_1^{i+2n-3}q'\theta}{1-q'2}\sigma^2.
\end{aligned}$$

Appendix D

Combining (1) and (6), we get

$$\begin{cases} Y_{it+1} = \alpha_1 - \xi_1 a_1 - \xi_1 \hat{\eta}_{i(t-1)+j} + \eta_{it+1}, \\ Y_{it+2} = \alpha_1 - \xi_1 a_1 - \xi_1 \hat{\eta}_{it+1} + \eta_{it+2}, \\ \vdots \\ Y_{it+j} = \alpha_1 - \xi_1 a_1 - \xi_1 \hat{\eta}_{it+j-1} + \eta_{it+j}. \end{cases}$$

Combining (7) and (14), we get

$$\begin{cases} Y_{it+1} = (1-\lambda\xi_1) Y_{i(t-1)+j} + \eta_{it+1} - \eta_{i(t-1)+j}, \\ Y_{it+n} = (1-\lambda\xi_1) Y_{it+n-1} + \eta_{it+n} - \eta_{it+n-1}, \end{cases}$$

$n = 2, \ldots, j$.

So

$$Y_{it+n} = (1-\lambda\xi_1)^{it+n} Y_0 + \sum_{k=0}^{it+n-1} (1-\lambda\xi_1)^k (\eta_{it+n-[k/j]^* i - k\%j} - \eta_{it+n-[k/j]^* i - k\%j - m}) = \sum_{k=0}^{it+n-1} q_1^k (\eta_{f(k)} - \eta_{f(k)-m'}).$$

Appendix E

According to Appendix D, assume $1 \leqslant n \leqslant j$, we get

$$\begin{aligned} Y_{it+i+n} = {} & q_1^j Y_{it+n} + \sum_{k=0}^{t} q_1^{jk} [(\delta + \varepsilon_{it+i+n-ki} - \theta\varepsilon_{it+i+n-1-ki}) \\ & + q_1(\delta + \varepsilon_{it+i+n-1-ki} - \theta\varepsilon_{it+i+n-2-ki}) + \cdots \\ & + q_1^{n-2}(\delta + \varepsilon_{it+i+2-ki} - \theta\varepsilon_{it+i+1-ki}) \\ & + q_1^{n-1}((i-j+1)\delta + \varepsilon_{it+i+1-ki} + (1-\theta)\varepsilon_{it+i-ki} + \cdots \\ & + (1-\theta)\varepsilon_{it+j+1-ki} - \theta\varepsilon_{it+j-ki}) q_1^n (\delta + \varepsilon_{it+j-ki} - \theta\varepsilon_{it+j-1-ki}) \\ & + \cdots + q_1^{j-1}(\delta + \varepsilon_{it+n+1-ki} - \theta\varepsilon_{it+n-ki})] \\ = {} & \sum_{k=0}^{t} q_1^{jk} \left[\left(q_1^{n-1}(i-j) + \frac{1-q_1^{j-1}}{1-q_1} \right) \delta + \varepsilon_{it+i+n-ki} \right. \\ & + \sum_{m=0}^{n-2} q_1^m (q_1-\theta)\varepsilon_{it+i+n-1-m-ki} \\ & + q_1^{n-1} \sum_{n=j+1}^{i} (1-\theta)\varepsilon_{it+n-ki} + \sum_{m=0}^{j-n-1} q_1 n \\ & \left. + m - 1 (q_1-\theta)\varepsilon_{it+j-m-ki} - q_1^{j-1}\theta\varepsilon_{it+1-ki} \right] \\ & \times \left[\varepsilon_{it+i+n-ki} + \sum_{m=0}^{n-2} q_1^m (q_1-\theta)\varepsilon_{it+i+n-1-m-ki} \right. \\ & + q_1^{n-1} \sum_{n=j+1}^{t} (1-\theta)\varepsilon_{it+n-ki} \\ & \left. + \sum_{m=0}^{j-n-1} (q_1-\theta)\varepsilon_{it+j-m-ki} - q_1^{j-1}\theta\varepsilon_{it+1-ki} \right] \\ & \times \left[\varepsilon_{it+i+n-ki} + \sum_{m=0}^{n-2} q_1^m (q_1-\theta)\varepsilon_{it+i+n-1-m-ki} \right. \\ & + q_1^{n-1} \sum_{n=j+1}^{i} (1-\theta)\varepsilon_{it+n-ki} \\ & \left. + \sum_{m=0}^{j-n-1} (q_1-\theta)\varepsilon_{it+j-m-ki} - q_1^{j-1}\theta\varepsilon_{it+1-ki} \right] \\ = {} & \begin{cases} \left[1 + \frac{(q_1-\theta)^2(1-q_1^{2j-2})}{1-q_1^2} + q_1^{2j-2}\theta^2 + q_1^{2n-2}(1-\theta)^2(i-j) \right]\sigma^2, \\ k' = k, \\ -q^{j-1}\theta\sigma^2, \ |k-k'| = 1, \\ 0, \ |k-k'| \geqslant 2. \end{cases} \end{aligned}$$

If $0 \leqslant q_1 < 1$, assume $S_{pd}(n) = \left[q_1^{n-1}(i-j) + \frac{1-q_1^{j-1}}{1-q_1} \right]^2$ and $S_{pi}(n) = 1 + \frac{(q_1-\theta)^2(1-q_1^{2j-2})}{1-q_1^2} + q_1^{2j-2}\theta^2 + q_1^{2n-2}(1-\theta)^2(i-j)$,

$$\begin{aligned} \text{AMSE}(Y_{it+i+n}) &= S_{pd}(n)\delta^2 + \sum_{k=0}^{t} q^{2kj} S_{pi}(n)\sigma^2 \\ &\quad - 2\sum_{k=0}^{t-2} q^{(2k+1)j}(\theta\sigma^2) \\ &= S_{pd}(n)\delta^2 + \frac{S_{pi}(n)}{1-q^{2j}}\sigma^2 - 2\frac{q^j}{1-q^{2j}} q^{j-1}\theta\sigma^2 \\ &= S_{pd}(n)\delta^2 + \frac{S_{pi}(n) - 2q^{2j-1}\theta}{1-q^{2j}}\sigma^2. \end{aligned}$$

Appendix F

$$\begin{aligned} \text{AMSE}_1 &= \frac{1}{j}\sum_{n=1}^{j} \text{AMSE}(Y_{it+i+n}) = \langle \text{AMSE}(Y_{it+i+n}) \rangle \\ &= \langle S_{pda}(n) \rangle \delta^2 + \frac{\langle S_{ps}(n) \rangle - 2q_1^{2j-1}\theta}{1-q_1^{2j}}\sigma^2. \end{aligned}$$

Appendix G

When $i = j = 1$, the plant becomes a single product plant. The AMSE is given by

$$\text{AMSE}_1(1,1) = \langle S_{pd} \rangle (1,1)\delta^2 + \frac{\langle S_{ps} \rangle (1,1) - 2\theta}{1-q_1}\sigma^2,$$

with

$$\langle S_{pd} \rangle (1,1) = \left(\frac{1}{1-q_1} \right)^2,$$

$$\langle S_{ps} \rangle (1,1) - 2\theta = 1 + \theta^2 - 2\theta.$$

Note that the above equation is equivalent to the expression given for single product control [4]:

$$\text{AMSE} = \frac{1+\theta^2 - 2\theta(1-\lambda\xi_1)}{\lambda\xi_1(2-\lambda\xi_1)}\sigma^2 + \left(\frac{\delta}{\lambda\xi_1} \right)^2.$$

It is obvious that

$$\begin{aligned}\langle S_{pd}\rangle(i,j) &= \left(\frac{1}{1-q_1}\right)^2 + \frac{(1-q_1^{2j})}{(1-q_1^j)^2(1-q_1^2)}\frac{(i-j)^2}{j} \\ &\quad + \frac{2}{(1-q_1)^2}\frac{i-j}{j} \\ &\geqslant \langle S_{pd}\rangle(1,1), \\ \langle S_{ps}\rangle(i,j) &= 1 + \frac{(q_1-\theta)^2(1-q_1^{2j-2})}{1-q_1^2} + q_1^{2j-2}\theta^2 \\ &\quad + \frac{(1-q_1^{2j})}{1-q_1^2}\frac{i-j}{j}(1-\theta)^2.\end{aligned}$$

Since

$$\begin{aligned}&(q_1-\theta)^2(1-q_1^{2j-2}) + (1+q_1^{2j-2}\theta^2 - 2q_1^{2j-1}\theta)(1-q_1^2) \\ &\quad = \theta^2 - 2q_1\theta - q_1^{2j} + 1 - q_1^{2j}\theta^2 + 2q_1^{2j+1}\theta \\ &\quad = (1+\theta^2-2q_1\theta)(1-q_1^{2j}), \\ &\frac{(q_1-\theta)^2(1-q_1^{2j-2})}{(1-q_1^{2j})(1-q_1^2)} + \frac{1+q_1^{2j-2}\theta^2-2q_1^{2j-1}\theta}{1-q_1^{2j}} = \frac{1+\theta^2-2q_1\theta}{1-q_1^2}, \\ &\frac{\langle S_{ps}\rangle(i,j) - 2q_1^{2j-1}\theta}{1-q_1^{2j}}\sigma^2 \\ &\quad = \frac{1}{1-q_1^{2j}}\left[1 + \frac{(q_1-\theta)^2(1-q_q^{2j-2})}{1-q_1^2} + q_1^{2j-2}\theta^2 \right. \\ &\qquad \left. + \frac{(1-q_1^{2j-2})}{1-q_1^2}\frac{(i-j)}{j}(1-\theta)^2 - 2q_1^{2j-1}\theta\right]\sigma^2 \\ &\quad \geqslant \frac{1}{1-q_1^{2j}}\left[\frac{(q_1-\theta)^2(1-q_1^{2j-2})}{1-q_1^2} + 1 + q_1^{2j-2}\theta^2 - 2q_1^{2j-1}\theta\right]\sigma^2 \\ &\quad = \frac{1+\theta^2-2q_1\theta}{1-q_1^2}\sigma^2 = \frac{\langle S_{ps}\rangle(1,1)}{1-q_1^2}\sigma^2, \\ &\mathrm{AMSE}_1(i,j) \geqslant \mathrm{AMSE}_1(1,1).\end{aligned}$$

Appendix H

As described in [18], the JADE control model is

$$y_k = bu_k + \hat{c}_{\mathrm{tot},k}.$$

The offset term $\hat{c}_{\mathrm{tot},k}$ is assumed to be a linear combination of contributions of different tools and different products. Let the total number of production contexts, e.g., different tools and different products, is q. In our simulation example, there are two tools and five products, $q = 7$. A production schedule for rth future runs, $[A_0]_{r\times q}$, can be defined. In $[A_0]_{r\times q}$ the tool used and product produced are specified by 1 in the relevant entry of each row, and 0s in the rest. If the contribution to the total bias is defined by $[c]_{q\times 1}$, the total bias estimated by

$$[\tilde{c}_{\mathrm{tot}}]_{r\times 1} = [A_0]_{r\times q}[c]_{q\times 1}.$$

The corresponding control actions are given by

$$[u]_{r\times 1} = \frac{[T]_{r\times 1} - [\tilde{c}_{\mathrm{tot}}]_{r\times 1}}{b}.$$

Similarly using a window of past data, the individual contributions of different tools and product $[c]_{q\times 1}$ can be estimated as the weighted least square solution of:

$$\begin{bmatrix} A_0 \\ I \end{bmatrix}_{(r+q)\times q}[c_{k+1}]_{q\times 1} = \begin{bmatrix} [y-bu]_{r\times 1} \\ c_k \end{bmatrix}_{(r+q)\times 1}.$$

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